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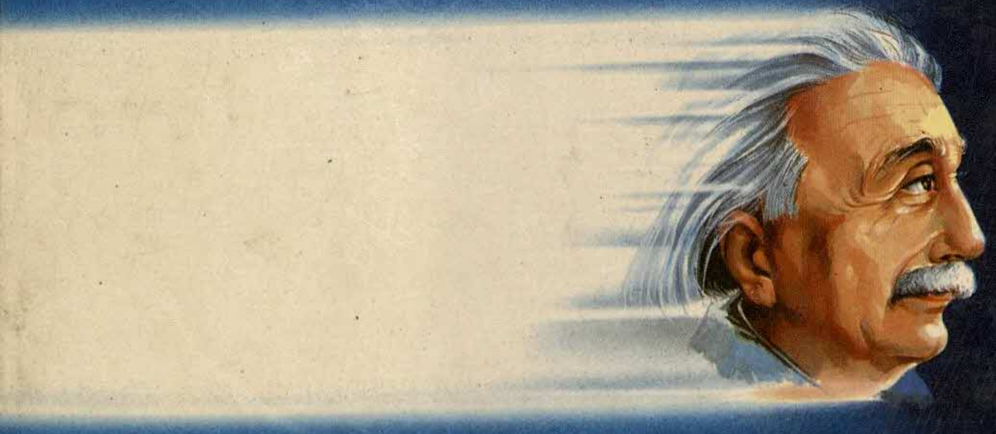
VIGNETTES IN PHYSICS

G. VENKATARAMAN

AT THE SPEED OF LIGHT



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At the Speed of Light

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At the Speed of Light

G. Venkataraman

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1.12.2010
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© Universities Press (India) Limited 1993

First published 1993
Reprinted 1994
ISBN 81 7371 009 0

Distributed by
Orient Longman Limited

Registered office
3-6-272, Himayatnagar, Hyderabad 500 029 (A.P.), India

Other offices
Kamani Marg, Ballard Estate, Bombay 400 038
17 Chittaranjan Avenue, Calcutta 700 072
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3-6-272 Himayatnagar, Hyderabad 500 029
28/31, 15 Ashok Marg, Lucknow 226 001
City Centre Ashok, Govind Mitra Road, Patna 800 004

Typeset by
Alliance Phototypesetters, Pondicherry 605 013

Printed in India by
Twin Cities Offset Printers, Hyderabad 500 029

Published by
Universities Press (India) Limited
3-5-820, Hyderguda, Hyderabad 500 029

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Preface

To the adult reader

This book and others in this series written by me are inspired by the memory of my son Suresh who left this world soon after completing school. Suresh and I often used to discuss physics. It was then that I introduced him to the celebrated *Feynman Lectures*.

Hans Bethe has described Feynman as the most original scientist of this century. To that perhaps may be added the statement that Feynman was also the most scintillating teacher of physics in this century.

The Feynman Lectures are great but they are at the textbook level and meant for serious reading. Moreover, they are a bit expensive, at least for the average Indian student. It seemed to me that there was scope for small books on diverse topics in physics which would stimulate interest, making at least some of our young students take up later a serious study of physics and reach for the Feynman as well as the Landau classics.

Small books inevitably remind me of Gamow's famous volumes. They were wonderful, and stimulated me to no small extent. Times have changed, physics has grown and we clearly need other books, though written in the same spirit.

In attempting these volumes, I have chosen a style of my own. I have come across many books on popular science where elaborate sentences often tend to obscure the scientific essence. I have therefore opted for simple English, and I don't make any apologies for it. If a simple style was good enough for the great Enrico Fermi, it is also good enough for me. I have also employed at times a chatty style. This is deliberate. Feynman uses this with consummate skill, and I have decided to follow in his footsteps (whether I have succeeded or not, is for readers to say). This book is meant to be read for fun and excitement. It is a book you can even lie down in bed and read, without going to sleep I hope!

Naturally I have some basic objectives, the most important of which is to stimulate the curiosity of the reader. Here and there the reader may fail to grasp some details, and in fact I have deliberately pitched things a bit high on occasions. But if the reader is able to experience at least in some small measure the *excitement* of science, then my purpose would have been achieved. Apart from excitement, I have also tried to convey that although we might draw boundaries and try to compartmentalise

Nature into different subjects, she herself knows no such boundaries. So we can always start anywhere, take a random walk and catch a good glimpse of Nature's glory. Where she is concerned, all topics are 'fashionable'. There is today an unnecessary polarization of the young towards subjects that are supposed to be fashionable. To my mind this is unhealthy, and I have tried to counter it.

This series is essentially meant for the curious. With humility, I would like to regard it as some sort of a 'Junior Feynman Series', if one might call it that. With much love, and sadness, it is dedicated to the memory of Suresh who inspired it.

To the young reader

This book is about the theory of relativity. In a sense, the story of relativity is largely the story of one man, Albert Einstein, who almost single-handed, created the theory. He started thinking about these matters even when he was a young boy and in 1905 (at the age of 26), proposed the Special Theory of Relativity. For many years, people could not understand this theory; only a few did. Why this was so you will learn later. In 1915, Einstein announced the General Theory of Relativity which was a natural successor to the Special Theory. It is being praised even today.

In this book, I shall be dealing mainly with the Special Theory. I trust that after reading the book you will find that the Special Theory is not all that difficult—at least as far as the basic ideas are concerned. I also hope you will get some idea of why Einstein became so famous.

Acknowledgments

I am deeply grateful to Mr. A. Ratnakar and Mr. John D. Vincent for helping me with the collection of background material. Thanks are due to Professor V. Balakrishnan, Professor N. Mukunda and Dr. M. V. Atre for a critical reading of the first draft. Mrs. Naga Nirmala rendered valuable assistance in the preparation of the manuscript. The cooperation of the publisher is also much appreciated.

G.VENKATARAMAN

1 *The Young Einstein*

Many discoveries seem sudden, but more often than not, great discoveries occur when the time is ripe for them. Even though one particular person might actually have made the discovery, we cannot overlook the preceding events or the scientists who contributed many of the ideas that in a sense shaped the discovery. As mentioned in the preface, the theory of relativity is the creation of Einstein—there is no question about that. But he did not quite produce it out of a vacuum. It all arose because there were some problems in tying up mechanics (as given to us by Newton) and the theory of electromagnetism (as proposed by Maxwell). Einstein pondered about these difficulties, and out of this effort was born the *Special Theory of Relativity*.

I am sure you have heard of Albert Einstein. But like most people, you probably don't know much about him. So let me tell you something about this great scientist. In this chapter I shall introduce to you the young Einstein; the later years are described in the last chapter.

Albert Einstein was born on March 15, 1879, in a small town named Ulm in Germany, to Hermann and Pauline Einstein. The Einsteins belonged to the Jewish faith, which created problems for Albert Einstein later. Besides Albert, Hermann and Pauline also had a daughter named Maria. Albert was much attached to his sister whom he called Maja.

Hermann Einstein was a businessman but not a very successful one. A few years after young Albert was born, Hermann moved to the city of Munich to start a new business with his younger brother Jakob, an engineer. In Munich young Albert received lessons in violin. In later years, the violin became Einstein's favourite musical instrument.

At the age of six, Albert Einstein entered the public school. In Germany it is called the Volksschule. In 1888, he moved from the Volksschule to the Luitpold Gymnasium where he studied until he was fifteen. The Gymnasium is the equivalent of our High School. Einstein did well in Latin and mathematics, but he disliked school. He became easily tired and did not care much for sports. He felt isolated and had few friends at school.

Albert liked Uncle Jakob very much because he would pose interesting mathematical problems. There was also a family friend named Max

Talmud who was a regular visitor at home. Talmud used to give young Albert books on popular science which the latter very much enjoyed. The two also used to discuss philosophy!

Hermann Einstein's business initially prospered but then began to fail. So he moved the factory to Milan in Italy. Albert was now left behind in Munich, and he felt miserable and alone. He managed to get a certificate from the family doctor in Munich, quit the Gymnasium, and headed for Italy! His parents were naturally very upset when one day they saw him at their front door in Italy. However, Albert assured them that he would study hard at home and prepare for the polytechnic in Zurich, Switzerland. In October 1895, Einstein went to Zurich to take the entrance exam. He failed, though he did well in maths and science. He then joined a tutorial school in Switzerland to prepare for the entrance test to the polytechnic Eidgenossische Technische Hochschule, simply called the ETH. This time he was successful in the entrance test, and studied at the ETH from 1896 to 1900. Einstein now began to enjoy science very much and resolved to make it his career. His interest in music and literature also grew. If you ever study the writings of Einstein, you will find them to be well composed.

At ETH, Einstein loved working in the lab. As he wrote later, "Most of the time I worked in the physical laboratory, fascinated by the direct contact with observation." However, he had some trouble with his professor. At one point this professor said to Einstein: "You are a smart boy, a very smart boy. But you have one great fault; you do not let yourself be told anything."

In 1900, Einstein passed the final examination with good grades in science. He was hoping that his professor would offer him an assistant's job, but that did not happen. Meanwhile, he published his first paper in the German journal *Annalen der Physik*. Job-hunting now started but without any success. For a short while he worked as a school teacher until in June 1902 he was appointed, on a temporary basis, as a technical expert, third class, in the patent office in the city of Berne (in Switzerland).

Einstein did well in the patent office. He took his work seriously, and even enjoyed it. He had a lot of spare time which he put to good use. How? By doing research, and by publishing original, scientific papers. In 1903 and 1904 he published papers on the foundations of statistical mechanics. The year 1905 was a memorable one for Einstein. It is also a memorable year for physics, for in that year he published three papers, each one a gem. People have said that every one of them deserved the Nobel Prize. All these papers were published in volume 17 of the *Annalen der Physik*. Starting on page 132 is the paper on the photoelectric effect.

Then on page 549 begins the paper on Brownian motion, and lastly on page 891 starts the paper on the Special Theory of Relativity. One of these papers eventually won for Einstein the Nobel Prize. People often think it is his work on relativity. Wrong! It is the paper on the photoelectric effect. Well, there is a reason why this happened but I shall not go into that here. Let me just say that this does *NOT* mean that the theory of relativity is inferior to, or less important than, the theory of the photoelectric effect. It is just that the Prize Committee had to chose one of them and it decided in favour of the photoelectric effect.

Meanwhile Einstein got married, and at the office he received a couple of promotions. In between all this, he also obtained his doctorate degree from the University of Zurich. In 1907, Einstein quit his job in the patent office to enter the academic world. We will pick up that part of the story later. For now we turn our attention to the Special Theory, the story of which is gradually unfolded in the following chapters.

2 *Newtonian Relativity*

2.1 Law of inertia

It was believed in ancient times that force is needed to sustain motion. On the face of it this seems correct, for, if we roll a ball on a flat surface, it soon comes to a stop. A constantly applied force, on the other hand, can keep it going.

Galileo (see Appendix) was the first to realise the mistake in this type of reasoning. Consider a flat table with a glass top on which is resting a nicely made steel cube. If the cube is pushed it would move some distance and come to a stop—no surprise in that. Suppose we generously spread a lubricant on the table and also apply it to the cube. This time the cube would travel a much longer distance before coming to a stop. If we use an even better lubricant, the cube would slide an even greater distance.

I am sure you have guessed what is happening. The cube comes to a halt because of the action of frictional forces. Lubricants reduce friction whence the cube is able to travel farther. What if we use a super lubricant that makes friction identically zero? Obviously then, the cube once given a push would keep sliding with a uniform speed. It does not matter that there is no such super lubricant. The point simply is that *force is not needed to sustain motion*.

Arguing along these lines, Galileo was led to the remarkable *law of inertia*:

> A body on which forces have ceased to act keeps moving with the same speed and in the same direction as it had at the instant when these forces ceased.

2.2 Enter Newton

Issac Newton (see Appendix) was born in the very year Galileo died. The coincidence is symbolically significant because Newton took off from where Galileo left.

Everyone is familiar with the famous laws of Newton. Let us focus on the first law which states:

> A body remains in a state of rest or of uniform motion in a straight line unless compelled to change that state by an applied force.

If you think about it for a minute, you will realise that this is simply Galileo's law of inertia but restated. This is the starting point of classical mechanics.

2.3 Absolute space and absolute time

College textbooks take you through Newton's laws and explain how to use them for solving problems about stones dropped, bullets fired, etc. Instead, I shall take you along a different route. Let us go back to Newton's first law and focus on two crucial phrases namely, *state of rest* and *uniform motion in a straight line.* What precisely does one mean by these phrases? In other words, *with respect to what* does the body remain at rest or in uniform motion? Unless one is able to answer this question, the law of inertia is not very meaningful.

Newton thought deeply about this problem and finally introduced two major, philosophical concepts, namely, *absolute space* and *absolute time.* About the former he says:

> Absolute space, in its own nature, without regard to anything external, remains always similar and immovable.

There are all kinds of problems with this absolute space, but I shall come to those questions later. About absolute time, Newton declared:

> Absolute, True, and Mathematical time, of itself, and from its own nature flows equably without regard to anything external ...

Later we shall see that there is no such thing as absolute time.

2.4 Principle of relativity—version I

Ignoring for a moment subtle points about absolute space, we can certainly say that rest and uniform motion could, at least in principle, be defined using the above concepts. Notice that the law of inertia treats a state of rest and a state of uniform motion on the same footing. What it means is that if there were two observers in absolute space, one of whom is at rest and the other is in uniform motion, Newton's laws and all conclusions derived therefrom would apply equally well to both observers. Formally, this is called the *principle of relativity*.

> The laws of mechanics in a frame of reference moving rectilinearly and uniformly through absolute space are exactly the same as in another frame which is at rest (in absolute space).

The jargon *frame of reference* would be explained shortly. But meanwhile, what the above words mean is simply this: Let O and O' be two observers, with O at rest (in absolute space) and O' moving with a uniform velocity (again in absolute space). There is an object (say a particle) in absolute space which is acted on by a force. Naturally, the particle would be accelerated—this is Newton's second law. O would describe it by saying $F = ma$ where as O' would say $F' = ma'$. Notice two important things: (i) The *form of the expression* is the same for both the observers, though the values of the forces and the accelerations might be different. (ii) The mass is the same for both.

2.5 The practical world

Einstein says, "The purpose of mechanics is to describe how bodies change their position in space with time." For example, this could refer to a falling stone, a flying bullet, etc. When we discuss a falling stone, we refer its height with respect to the local ground level—no messing around with absolute space or anything like that. In fact, most of our school and college teachers are not aware that Newton actually started with absolute space. The genius that he was, Newton realised that in practical problems one is always concerned with *relative* motion.

A convenient and detailed scheme for discussing relative motion emerged after Ludwig Lange introduced what is called the *inertial reference frame* or *inertial frame* for short. Before proceeding further, let me explain what I mean by a frame of reference. In simple language, it just means a suitable set of coordinate axes. For example, places on the globe are located by latitude and longitude. In physics it is customary to choose cartesian axes as in Fig. 2.1. The axes provide a reference frame with respect to which one could describe the motion of particles etc. How

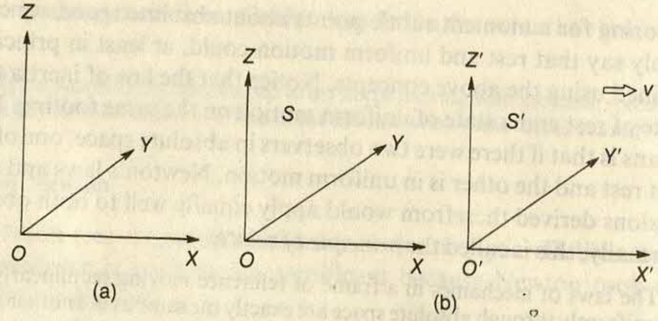


Fig. 2.1 (a) Cartesian axes. (b) Two frames S and S', represented respectively by the axes OX, OY, OZ and $O'X'$, $O'Y'$, $O'Z'$. S' moves with a uniform velocity v with respect to S in the direction shown.

exactly such a description is given will be explained later. *An inertial reference frame is one on which no forces are acting (i.e., it is not accelerating). Therefore, in this frame, Newton's first law would hold.*

Can one really have, in practice, a frame on which *no forces whatsoever* are acting? Not really, but then in many situations the forces acting may either be negligible or produce effects which are hardly observable on the time scale of observation. For example, our Earth rotates about its own axis and also goes around the Sun. Clearly there are accelerating forces due to all this and yet for many purposes, it is quite a good approximation to treat a coordinate frame, rigidly attached to the Earth's surface, as an inertial frame. In fact, in our usual "stone falling down" problems, this is precisely what we are doing (though implicitly)—see also Box 2.1.

Box 2.1 In a book entitled *The Evolution of Physics*, Einstein and Infeld discuss the question of the inertial system using an imaginary dialogue.

> Let us interview the classical physicist and ask him some simple questions:
>
> *"What is an inertial system?"*
>
> "It is a CS [coordinate system or what we would call a frame of reference] in which the laws of mechanics are valid. A body on which no external forces are acting moves uniformly in such a CS. This property thus enables us to distinguish an inertial CS from any other."
>
> *"But what does it mean to say that no forces are acting on a body?"*
>
> "It simply means that the body moves uniformly in an inertial CS."
>
> Here we could once more put the question: "What is an inertial CS?" But since there is little hope of obtaining an answer differing from the above, let us try to gain some concrete information by changing the question:
>
> *"Is a CS rigidly connected with the earth an inertial one?"*
>
> "No, because the laws of mechanics are not rigorously valid on the earth, due to its rotation. A CS rigidly connected with the sun can be regarded for many problems as an inertial CS; but when we speak of the rotating sun, we again understand that a CS connected with it cannot be regarded as strictly inertial."
>
> *"Then what, concretely, is your inertial CS, and how is its state of motion to be chosen?"*
>
> "It is merely a useful fiction and I have no idea how to realize it. If I could only get far away from all material bodies and free myself from all external influences, my CS would then be inertial."
>
> *"But what do you mean by a CS free from all external influences?"*
>
> "I mean that the CS is inertial."
>
> Once more we are back at our initial question!

2.6 Principle of relativity—version II

Suppose we have an inertial frame S. Let S' be another inertial frame moving with a uniform velocity with respect to S—see Fig. 2.1(b). Given these, the principle of relativity can now be restated as follows:

> The laws of mechanics remain the same for observers in inertial frames that are in uniform motion with respect to each other.

Mechanics as originally formulated by Newton was based on absolute motion. At the same time, Newton was realistic enough to remark, "And so instead of absolute places and motions, we use relative ones ..." For him, absolute space was sacred but not so useful whereas "relative space" was mundane but more useful. Newton maintained a disdainful attitude towards relative motion but later workers realised that relative rather than absolute motion is all that can be observed and is useful in mechanics. What is now commonly called *Newtonian mechanics* is based on relative motion.

2.7 Galilean transformations

Galileo considered a question asked a long time ago by Ptolemy, namely, "What happens when a stone is thrown up?" According to Ptolemy, due to the rotation of the Earth the stone would fall in a place different from where it was thrown—see Fig. 2.2. Galileo thought this was nonsense

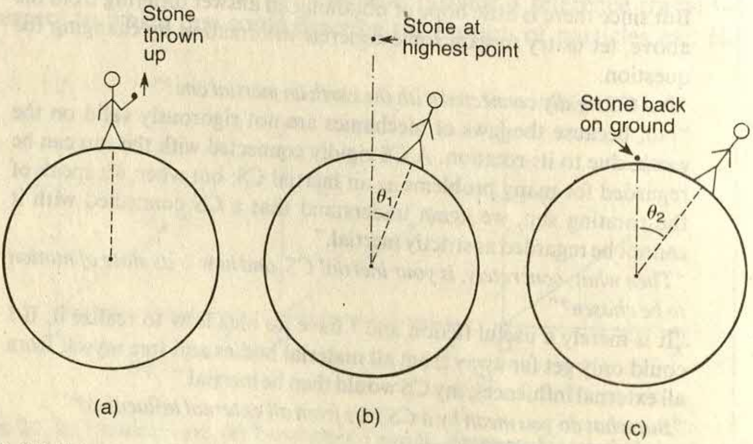


Fig. 2.2 Suppose a stone is thrown up as in (a). Ptolemy thought that on account of the rotation of the Earth, the stone would fall elsewhere. His idea is illustrated in (b) and (c).

and reasoned differently. He analysed what would happen when one drops a stone from the mast of a moving ship—see Fig. 2.3—and came to the conclusion that the stone would fall at the bottom of the mast. Ptolemy, on the other hand, would have argued that the stone would fall elsewhere and could even fall into the water provided the ship travelled fast enough. We know Galileo is right because while travelling in trains we do occasionally drop things and they fall right near our feet. On the other hand, for a person outside the train and standing by the track, the end point of the falling object would not be vertically below the starting point. So the description of the fall as given by two observers, one riding in the train and the other who is stationary and outside the train, would vary. But clearly there must be a relation between the two descriptions. There is, and it is called the *Galilean transformation*.

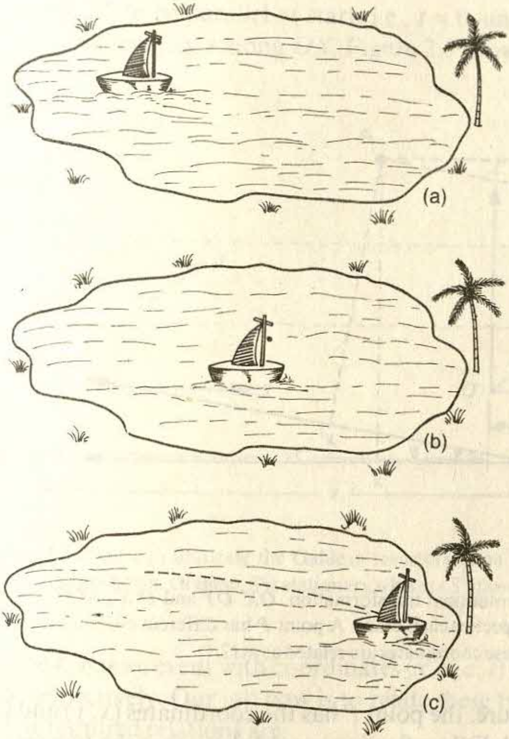


Fig. 2.3 Galileo considered the problem of a stone dropped from the mast of a moving boat—see (a). He reasoned that to an observer in the boat, the stone would appear to fall at the foot of the mast—see (b) and (c)—whereas to an observer on the shore the stone would appear to follow the dotted line in (c).

Transformation means change. In mathematics there are all kinds of transformations. Here we are dealing with physics and trying to relate the description of the *same* event by two observers in inertial frames which are moving uniformly with respect to each other. But before considering this problem, the following simple example might be useful. Figure 2.4 shows two sets of axes, one rotated with respect to the other.

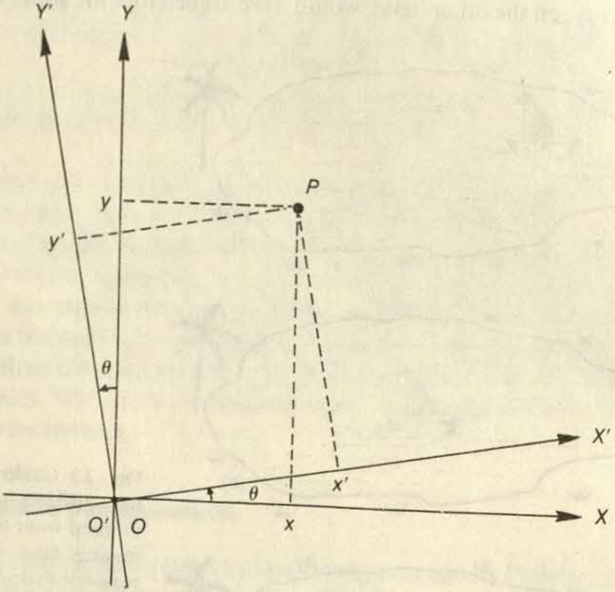


Fig. 2.4 Illustration of the rotational transformation. OX, OY and $O'X'$, $O'Y'$ are two sets of axes rotated with respect to each other. A point P has different coordinates in the two frames of references. These coordinates are related as in (2.1).

As is clear from the figure, the point P has the coordinates (x, y) and (x', y') in the two frames. A little reflection would show that the two sets of coordinates are related by the following relations:

$$\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned} \tag{2.1a}$$

$$\begin{aligned} x &= x'\cos\theta - y'\sin\theta \\ y &= x'\sin\theta + y'\cos\theta. \end{aligned} \tag{2.1b}$$

In short, (2.1) enables us to relate (x, y) and (x', y').

What is it that we wish to relate using Galilean transformations? Events. In normal usage the word *event* means an occurrence—it could be an accident, a cricket match or an election. In mechanics (and in relativity), an event refers to a happening *at a particular point in space at a particular instant of time*. So an event must be specified by the coordinates (x, y, z)

of the occurrence and also by the time t. In short we must specify the *space-time* coordinates (x, y, z, t).

Consider now two inertial frames S and S'. OX, OY, OZ denote the coordinate axes of the former and OX', OY', OZ' those of the latter. For convenience, let us assume that S is stationary. We align the axes of S and S' to be parallel at start, i.e., $t = 0$, and then let S' move with a uniform velocity v along OX. Figure 2.5 shows the two frames at some time t. P is an event with coordinates (x, y, z, t) and (x', y', z', t') in S and S' respectively. Our job now is to relate these two sets rather as in (2.1). The required relations are:

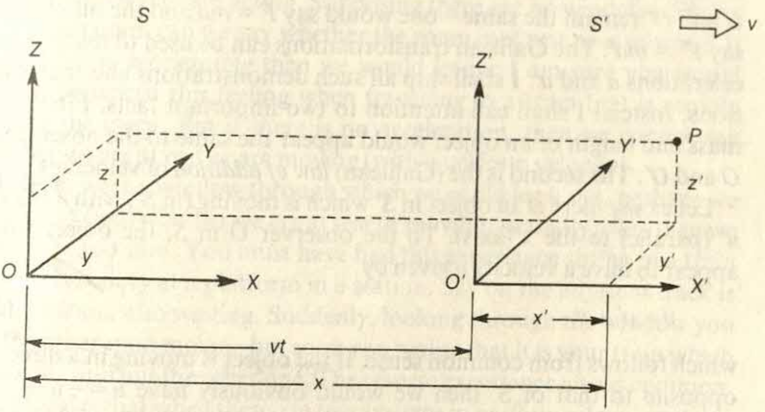


Fig. 2.5 Figure to illustrate the Galilean transformation (2.2). S and S' are two frames oriented as shown. Of these, S is stationary whereas S' moves with a uniform velocity in the direction OX.

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned} \tag{2.2a}$$

$$\begin{aligned} x &= x' + vt \\ y &= y' \\ z &= z' \\ t &= t'. \end{aligned} \tag{2.2b}$$

These rules are simple and follow from common sense. Notice the innocent looking statement $t' = t$. It simply means that clocks behave the same way in the two inertial frames. It is this statement which Einstein questioned on the road to his famous discovery.

2.8 Some consequences

The consequences of the Galilean transformations are many and I cannot discuss them here. The essential point is that given two frames S and S' as in Fig. 2.5, the *forms* of the equations of mechanics for observers at O and O' remain the same—one would say $F = ma$ and the other would say $F' = ma'$. The Galilean transformations can be used to relate the accelerations a and a'. I shall skip all such demonstrations and manipulations. Instead I shall call attention to two important facts. Firstly, the mass and length of an object would appear the same to the observers at O and O'. The second is the (Galilean) *law of addition* of velocities.

Let us say there is an object in S' which is moving (in S') with a velocity u' (parallel to the x'-axis). To the observer O in S, the object would appear to have a velocity u given by

$$u = u' + v \tag{2.3}$$

which follows from common sense. If the object is moving in a direction opposite to that of S′ then we would obviously have $u = -u' + v$. In short, the velocities add like vectors. A typical illustration of this rule is given in Fig. 2.6.

I remarked that rule (2.3) follows from common sense. You will soon

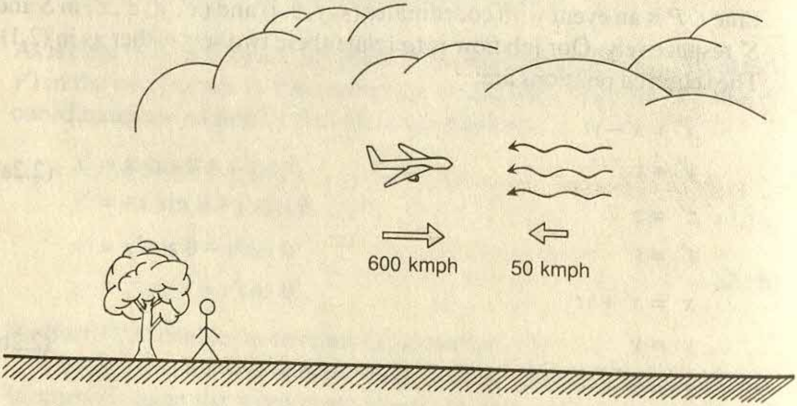


Fig. 2.6 A practical illustration of formula (2.3) A plane is flying at a speed of 600 kmph. This is the speed with respect to the surrounding air. If there is a wind (called headwind) blowing as shown, an observer on the ground would see the plane move with an effective speed of 550 kmph.

discover that common sense makes little sense once *v* becomes nearly equal to *c*, the velocity of light!

2.9 Who is moving?

Let us say we are in an inertial system. It could simply be a room far out in space where one might assume (at least to a very good approximation) that no forces are acting. Supposing there are no windows. Sitting inside the room, can we say whether the room is at rest or is moving? If the system is *accelerating* then we would know; I am sure you would have experienced this feeling when travelling in a train that is rapidly picking up speed. But if there is no acceleration, then we cannot say whether we are at rest or are moving (with a uniform velocity).

If there were a window through which we could look out, perhaps we could make out whether we are at rest or moving. Actually, even then we cannot be too sure. You must have had this experience sitting in a train that is stationary at a platform in a station. Say on the adjacent track is another train, also waiting. Suddenly, looking through the window you see the other train moving but soon you realise that it is your train which is moving and not the other one. The reverse experience also is common. The moral is that when there are two systems in *uniform relative* motion, it is not easy to say which one is at rest and which one is moving. This is not to suggest that uniform motion cannot be detected. For example, out there in space there are what are called *distant fixed stars*. So, if our room has a window and looking through it, it appears that the stars are moving, then it really means that *we* are moving. However, *without referring to something external*, an observer in an inertial frame *cannot* say whether he is moving or not. More important, *from a physics point of view, between two systems in uniform relative motion, it does not really matter which system is at rest and which one is moving*.

2.10 Pseudo forces

I said an inertial frame is one where no forces are acting. In practice, it is difficult to realise such a frame. Then what does one do?

Consider two frames *S* and *S′* as in Fig. 2.7. Of these, we shall assume *S* is stationary. *S′* not only moves with respect to *S* but also tumbles. And a particle is moving in *S′*! Obviously, things are pretty complicated in *S′*. This is not an idealised example but a real one. *S* could be a set of axes tied to the Sun (which here we assume to be stationary). *S′* could be a set of axes rigidly tied to the surface of the Earth. The Earth rotates about an axis and moves in space around the Sun. So our example is a

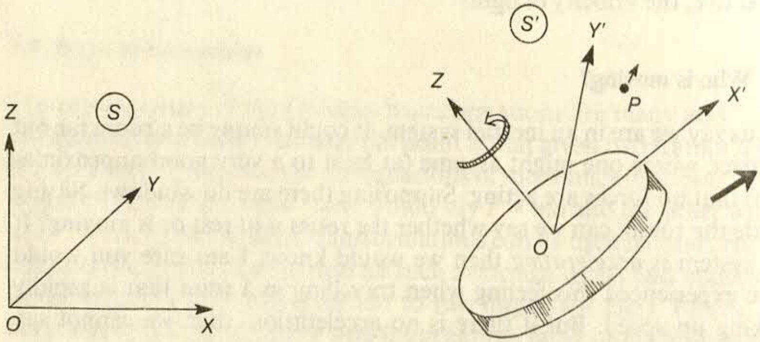


Fig. 2.7 S is a stationary reference frame. S' is a frame that is not only accelerating (in the direction of the bold arrow) but also rotating with respect to S. P is a particle moving in S'; altogether, a complicated situation! In this case, even if no force is applied on P, there are various kinds of pseudo forces acting on it. When S' moves with a uniform velocity, instead of tumbling etc., all the pseudo forces vanish.

meaningful one. Suppose a force F_{app} is applied to the particle. Can we, as we normally are used to do, write $F_{\text{app}} = \text{mass} \times \text{acceleration}$, the acceleration being described with respect to S'? No, because additional forces called *pseudo forces* are present which arise on account of the various complications mentioned above. Therefore, we have instead

$$\text{mass} \times \text{accln.} = F_{\text{app}} + F_{\text{pseudo}}. \tag{2.4}$$

However, if F_{pseudo} is negligible compared to F_{app}, then

$$\text{mass} \times \text{accln.} \sim F_{\text{app}}$$

which is the rule obeyed in an inertial frame. So whenever we treat the Earth as an inertial frame (which we often do), it is clearly an approximation. Depending on the sensitivity of our apparatus and the accuracy of our measurements, it might even be a good approximation.

2.11 Free fall

Consider Fig. 2.8(a). This shows a lift in which the rope has snapped. As a result, the lift falls freely. Such a system is often discussed in books on relativity, and the reason for this can be better understood from Figs. 2.8 (b) and (c). In (b), we have an observer (tough luck for him!) releasing a ball. Since the ball and the lift are both freely falling, the ball appears to "float" as illustrated in (c). Thus, to the observer in the lift, it would

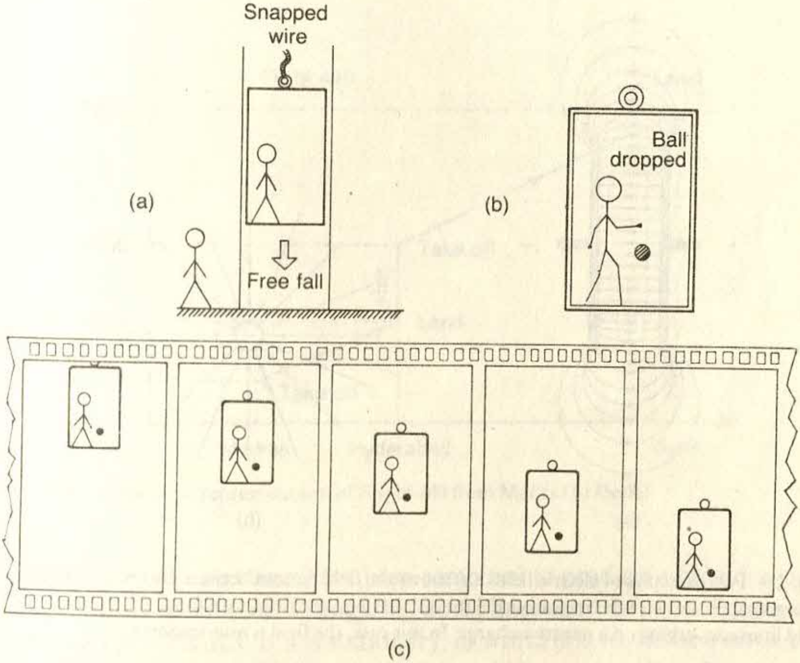


Fig. 2.8 (a) Schematic of a freely-falling lift. (b) shows an observer in the lift releasing a ball. Since both the lift and the ball are freely falling, the ball appears to float (to the observer inside). (c) shows a motion picture of the lift, the observer, and the ball, all falling together.

seem as if no forces are acting on the ball. In fact, using the ball he could verify Newton's first law or in other words, the freely-falling lift is an inertial frame *as far as the observer inside is concerned.*

Supposing there is another observer outside who is watching the lift crashing down. He would actually see the lift accelerating. Hence the observer inside and outside are *NOT* in *uniform* motion relative to each other. So they cannot relate events via Galilean transformations.

Let us now examine the freely-falling lift a bit more closely and see under what conditions the observer inside may regard it as an inertial system. An important requirement is that the gravitational field acting on the system must be *uniform*. The meaning of the word uniform may be understood from Fig. 2.9. Essentially, field lines should not crowd in some regions and be sparse in others.

As in the case of the electrostatic field (illustrated in Fig. 2.9), the gravitational field of the Earth is not uniform over large distances. But if

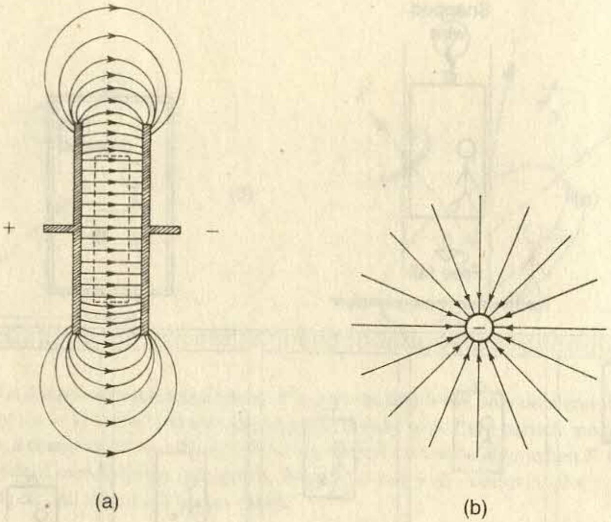


Fig. 2.9 Two examples of electric fields. (a) shows the field pattern between two metal plates charged oppositely. In the region within the dotted lines, the field is uniform. (b) shows the field lines converging on a negative charge. In this case, the field is non-uniform.

the lift is small and is falling through a short distance then the field acting may be considered sensibly uniform. We shall come back again to free fall in the last chapter when we consider the General Theory of Relativity.

2.12 Space-time and world lines

Earlier I told you what is meant by an event. Often we are interested in a sequence of events. Such a locus is referred to as a *world line*. And to represent the world line, we need a space-time diagram.

The idea of the world line can be understood from Fig. 2.10 which illustrates the events connected with Flight 440 from Madras to Delhi. The flight particulars are as follows:

Madras	departure	:	0600
Hyderabad	arrival	:	0650
	departure	:	0750
Delhi	arrival	:	0950

The space-time diagrams used in relativity follow the idea illustrated in Fig. 2.10. Two examples of these are shown in Fig. 2.11(a). Here one

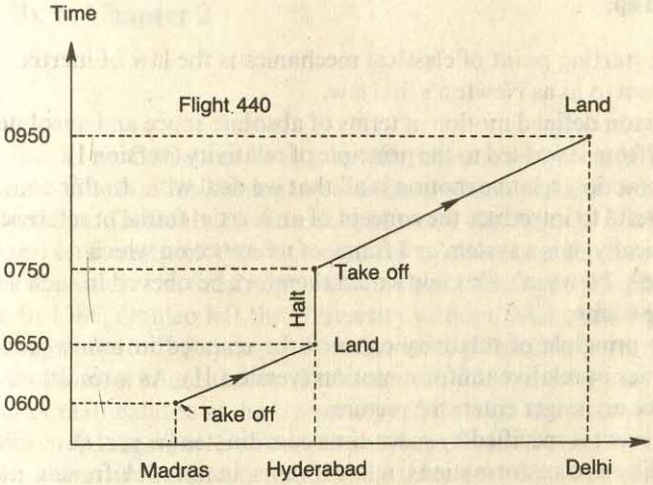


Fig. 2.10 A pictorial representation of Flight 440 from Madras to Delhi.

space-dimension and time have been combined to produce a hybrid two-dimensional space-time. Using this, world lines can be drawn. Suppose there is a particle at x. If it is stationary, its world line would be a vertical line as in Fig. 2.11(b); if it is moving, it would be a curved line. Later, we shall encounter world lines frequently.

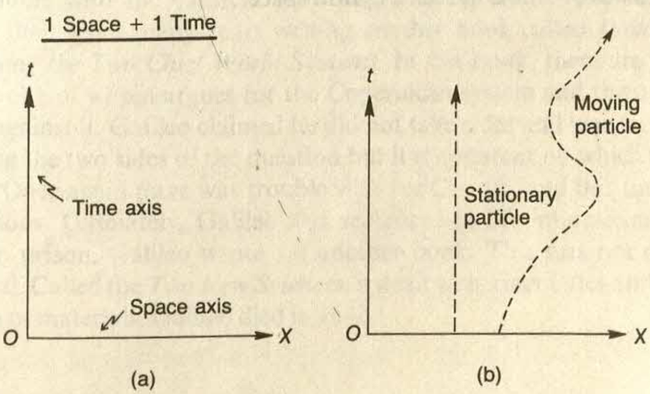


Fig. 2.11 (a) Space-time obtained by combining one space dimension with time. (b) World lines of a stationary particle and of a moving particle. Compare with Fig. 2.10.

To sum up:

- The starting point of classical mechanics is the law of inertia, better known to us as Newton's first law.
- Newton defined motion in terms of absolute space and absolute time and from it was led to the principle of relativity (version I).
- In practice, relative motion is all that we deal with. In this context, it is useful to introduce the concept of an inertial frame of reference.
- Basically, it is a system or a frame of reference on which no forces are acting. Newton's first law would therefore be obeyed in such a reference frame.
- The principle of relativity can now be restated in terms of inertial frames in relative uniform motion (version II). As a result, absolute space no longer enters the picture.
- An event is specified by space-time coordinates (x, y, z, t).
- Galilean transformations relate events in inertial frames moving uniformly relative to each other.
- An important fall-out is the law of addition of velocities stated in (2.3).
- It is not possible for an observer in an inertial system to determine his velocity without referring to something external.
- Pseudo forces are present in accelerated frames of reference. In such cases, Newton's second law must be used with care.
- A freely-falling lift would appear as an inertial system to an observer inside, provided the gravitational field across the lift is sufficiently uniform during the entire fall.
- Space-time is a hybrid space combining space and time. A world line is a locus of events in such a hybrid space.

Appendix to Chapter 2

Galileo

Galileo Galilei or Galileo as he is generally known, was born in Pisa, Italy in 1564. The famous leaning tower is situated here. Galileo's father was a musician. Galileo was the eldest of seven children. In 1581, he entered the University of Pisa as a medical student, but mathematics attracted him more. His father opposed Galileo taking interest in mathematics. In 1585, Galileo left the University without taking his degree. He then began to study physics and mathematics privately. During this period he became interested in mechanics. In 1589, he became the Professor of mathematics at the University of Pisa, and wrote a book on mechanics called *De motu.* Galileo was poorly paid, and he in turn made fun of the university administrators. So his three-year contract was not renewed, and he moved to the University of Padua. Here he not only wrote books but also designed an intricate compass which was of use to engineers and military men. He set up a small workshop to manufacture these compasses. While on a visit to Venice in 1609, Galileo learnt that a Dutch lens grinder named Hans Liperhey had applied for a patent for a device which would make distant objects appear closer. Galileo then started making telescopes, and built many of them. His telescopes were good, and using them he discovered the craters on the moon as well as the moons of Jupiter. In a book called *Letters on Sunspots* published in 1613, Galileo supported the Copernican system which promptly got him into trouble with the Church. However, nothing serious happened. Galileo then spent many years writing another book called *Dialogue Concerning the Two Chief World Systems.* In this book, there are two persons one of whom argues for the Copernican system and the other argues against it. Galileo claimed he did not take sides and was merely reporting the two sides of the question but it is apparent on which side he was. Once again there was trouble with the Church, and this time it was serious. Ultimately, Galileo was sentenced to life imprisonment. While in prison, Galileo wrote yet another book. This was not controversial. Called the *Two New Sciences,* it dealt with kinematics and the strength of materials. Galileo died in 1642.

Newton

Issac Newton was born on Christmas day in 1642. He was a premature baby, and it was said that at birth he could have fitted into a small mug!

Newton's father died before he was born, and his mother married again. Newton did not like his step-father.

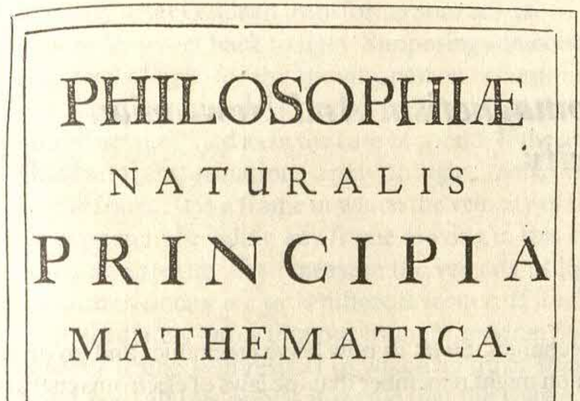
Newton came from a family of farmers but wasn't at all interested in farming. Instead, as a boy he often kept himself busy by building mechanical gadgets. He showed much interest in studies, and his family agreed that he could enter the University. Newton joined the Trinity College in Cambridge, and obtained the B.A. degree in 1665. Just about that time, there was an epidemic of plague, and the University was closed. Newton then went to the countryside where for eighteen months he worked on mathematics, optics, mechanics and astronomy.

In 1667, the University was reopened, and Newton was back. Two years later, he was appointed the Lucasian Professor. The professorship exists even today, and is held by very distinguished people. The Lucasian Professor was supposed to give at least one lecture every week during the term. Apparently, Newton's lectures were not very popular, and sometimes there was no audience! So Newton spent most of his time in his room working very hard and making discoveries.

I will not attempt to describe within a brief Appendix, Newton's monumental contributions. Any attempt to do so would be an insult to him. Let me just say that he has made remarkable contributions to mathematics, optics, mechanics and astronomy. And he will be remembered for many things, including his famous book the *Principia*.

Newton was not merely a scientist. He was closely associated with the Royal Society and later became its President. He was a Member of Parliament, and was also appointed as the Master of the Mint (the place where coins are made). Newton was a friend of Halley (after whom the famous comet has been named) and the astronomer Flamsteed. Newton used to employ the data of Flamsteed to check out his calculations about celestial bodies. At one stage, Halley and Flamsteed quarrelled, and Newton took the side of Halley. Thereafter, Flamsteed refused to supply his observations to Newton. In 1712, Newton and Halley produced a book containing some observations of Flamsteed. They did not take Flamsteed's permission, saying that since Falmsteed was being paid by the public, his research was also public property! Flamsteed went to court and won, and had the pleasure of burning all the copies of the work Newton had published.

Newton also had a long battle with the German mathematician Leibnitz as to who had invented calculus first. Newton charged that Leibnitz had stolen it from him (i.e., Newton). Leibnitz then appealed to the Royal Society for a hearing but Newton being a member of the Society, managed to make sure that the Committee which heard



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *JS. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos
Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtant Venales apud *Sam. Smith* ad inſignia Principis *Walliæ* in Cœmiterio
D. *Pauli*, alioſq; nonnullos Bibliopolas. *Anno* MDCLXXXVII.

Title page of the first edition of Newton's *Principia*.

Library
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Leibnitz's complaint comprised of his (Newton's) friends. In fact, Newton himself wrote the Committee's report! In short, while Newton was undoubtedly one of the greatest scientists of all time, as a person he was far from straight.

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3 *Electromagnetism And Newtonian Relativity*

3.1 Light and ether

Physics is not all mechanics. So let us now leave mechanics and go on to electromagnetism. You might remember that the laws of electromagnetism were discovered due to the efforts principally of Cavendish, Faraday, Ampere and Maxwell, the last of whom wrote down the famous equations of electromagnetism, now appropriately called *Maxwell's equations*.

According to Maxwell, electromagnetic radiation travels at a speed c equal to 3×10^{10} cm/sec. Prior to Maxwell's work, the speed of light had already been determined experimentally and the measured value was in close agreement with the value calculated for c by Maxwell. This clearly established that light was nothing but electromagnetic radiation, and this was one of the great successes of Maxwell's theory.

Now the question may be asked: "With respect to *what* does light travel at a speed of 3×10^{10} cm/sec ?" The question is not odd, for when we say that sound travels at a speed of 330 m/sec, what we really mean is that sound travels in *air* at this speed. Same way, in those days it was believed that light travelled in a mysterious medium called the *ether*, and which had strange properties. It was supposed to be perfectly transparent, have zero density and penetrate everything including a piece of solid steel. So the speed of 3×10^{10} cm/sec was supposed to be the speed of light in *stationary* ether.

3.2 Is there an absolute frame?

This immediately made people wonder about what happens when the ether is moving or the light source is moved with respect to the ether. To get a feel for this problem, consider a siren fixed to the roof of a factory. If the speed of sound in air is v and we are standing at a distance L from the siren, then sound from the siren would take a time (L/v) to reach us. If we travel in a car towards the siren, the sound would reach us earlier since we are going ahead to "meet" it. In other words, if v_c is the speed of the car, sound would now appear to travel with a speed ($v + v_c$). This is precisely what Galilean transformations tell us.

Now let us get back to light. Supposing someone moves in ether. Will the speed of light for this moving person become modified as in the case of sound? In those days, people believed that the speed of light would indeed get modified as in the case of sound. If the above is true, i.e., if the Galilean transformations apply to light, then ether at rest becomes a *unique* frame. It is a frame in which the velocity of light is exactly c and if we happen to be riding any frame moving in this ether, then we can, by doing an experiment to measure the velocity of light in our frame find out if the velocity is c or is different from c. If it is different from c then we immediately know that our frame is moving. So the ether at rest is an *absolute* frame with respect to which all other frames can be described. Mind you, all this only if it is true that the velocity of light depends on the speed of the frame in which it is measured.

The crucial experiment, to check whether the velocity of light varies with the speed of the observer, was done in the second half of last century. More about it shortly.

3.3 Maxwell's equations and the Galilean transformations

Let us now look at Maxwell's equations from a slightly different point of view by asking the question: "The laws of (Newtonian) mechanics remain the same under the Galilean transformations. Do the laws of electromagnetism behave similarly?" This is a fair question because *we want to make sure that the laws of electromagnetism are the same for observers in different inertial frames*. This is something we *must* have if we want to hang on to the principle of relativity. When one studies such questions, one usually uses words like *invariance* and *covariance*, which are explained in Box 3.1.

Box 3.1 This box deals with the concept of *invariance* and *covariance*. One can write a big chapter on these topics, but I shall be brief and try to keep things simple. Roughly speaking, when something does not change, one says it is invariant. The concept of invariance is very important and is used extensively in physics, but one must be careful. When one says something is invariant, one should specify: Invariant with respect to what? For example, when we calculate the energy of a hydrogen atom, say using the Bohr model, the energy comes out to be the same no matter how we choose the coordinate axes for describing the orbiting electron. We would then say that the energy of the hydrogen atom is invariant with respect to the choice of the coordinate axes used in the calculation.

The kind of invariance we are concerned with in relativity is the invariance of physical laws. For example, you must have heard about the conservation of momentum. If two billiard balls collide, then (provided there is no friction etc.), the total momentum of the two balls before and after the collision remains the same—this is the law of conservation of momentum. We expect this law to hold in all inertial frames, and when we transform from one inertial frame to another. We therefore say that the law of conservation of momentum in collisions is invariant under a transformation from one inertial frame to another. It is not merely laws that remain invariant; depending on what they are, certain physical quantities also remain invariant. For example, the charge of an electron remains invariant in the same sense as the momentum conservation law.

Invariance is important no doubt but not the only thing of interest. This brings me to *covariance*. This term is usually applied to equations. What it means is that there is *no change in form*. Without going into technical details, I can explain the spirit of the idea.

Let us consider the electric field **E** and the magnetic induction **B**. As you know, these are vectors, i.e., **E** has components E_x, E_y, E_z and similarly does **B**. Say **E** and **B** are defined in an inertial frame S. Given below is a sample of Maxwell's equations (in component form).

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}. \qquad (1)$$

Doesn't matter if you don't quite follow all this; just concentrate on the *form* of the above equation. Now let us consider an inertial frame S' which is moving uniformly with respect to S with a velocity v. In this frame, the electric field **E** and the induction **B** would be different—say they are described by the vectors **E′** and **B′**.

The job now is to relate **E** and **B** with **E′** and **B′** and then examine the forms of the equations for **E′** and **B′**.

Things don't work out if Galilean transformations are used. If instead one uses *Lorentz transformations* (to be described in Chapter 5), one finds the analogue of (1) to be:

$$\frac{\partial E'_z}{\partial y'} - \frac{\partial E'_y}{\partial z'} = -\frac{\partial B'_x}{\partial t'}. \qquad (2)$$

Observe that (1) and (2) have the *same form*. This is what is meant by covariance.

Going back to the question I just raised, it turns out that the form of Maxwell's equations get *changed* under Galilean transformations. This is bad news, and it could mean one of three things:

1. The principle of relativity does not apply to electromagnetism. In other words, all inertial frames are *NOT* equivalent, and there

could be a preferred frame. As you know, the etherwallas were already wondering whether the ether at rest was such a frame.

2. The problem is with Maxwell's equations; that is why they are not covariant.
3. The principle of relativity actually applies both to mechanics as well as electromagnetism. The problem is not with Maxwell's equations but with Galilean transformations which are supposed to guarantee relativity.

For a while, everything looked very puzzling. Maxwell's equations were so good that it seemed a crime to suspect them. Galilean transformations also seemed solid like a rock. Could it be that the principle of relativity was a fiction? Maybe there *was* a preferred or an absolute frame.

3.4 The hunt for an absolute frame

Let us follow this idea about an absolute frame a bit further. Now when the Earth goes round the Sun, there must be an ether wind just as we experience a breeze while travelling in an open car. In fact, this ether wind must have a speed of about 30 km/sec, which is the speed of Earth's orbital motion around the Sun. Albert Michelson (about whom you will hear more in the next chapter) decided to look for this wind, or rather the effect which an ether wind would have on the velocity of light. He did the first of such experiments, in 1881. In 1889, he repeated it much more carefully along with another American scientist E.W. Morley—this is the famous Michelson–Morley experiment about which I shall give you some historical information in the next chapter. Here let me discuss the principle of the experiment.

Consider first Fig. 3.1(a) which shows a lake. There is no wind, whence the water is absolutely still. As there is no wind, it is equally easy to row in any direction. Thus the time taken for a person to row the boat 50 metres to the east from the centre, would be the same as the time to row it 50 metres to the north, provided the rowing speed v is the same.

In Fig. 3.1(b) we have a river. As before, the man in the boat rows it at a certain speed v say, first a distance L downstream and then again a distance L, this time perpendicular to the current. In both trips, he maintains the same rowing speed. Nevertheless, on account of the current in the river, the two trips would take different lengths of time.

Michelson's idea was very similar. He argued that the ether wind is like the current in the river; and so, one could send a light beam down (or even against) this current over some distance L, and then send the

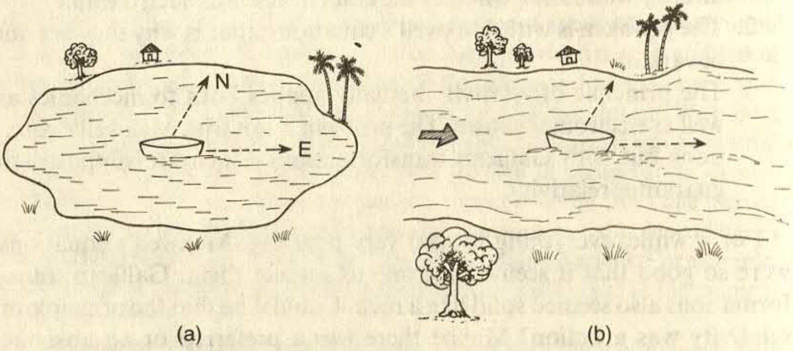


Fig. 3.1 (a) shows a boat in a still lake. If the same rowing speed is maintained, there should be no difference in the time taken to row 50 metres from the centre in an easterly direction, and to row 50 metres towards the north. In (b), the boat is now in a river. If the current is strong, obviously rowing downstream would be much easier than going across. Thus, maintaining the *same rowing speed*, it should take less time to go 50 metres downstream than 50 metres across.

beam again over the same distance but this time perpendicular to the current. If the time taken for these two trips is different, then there is an ether wind. If the time taken in both cases is the same, then there is no ether wind.

Michelson did not quite measure the travel time; he did something equivalent. The principle of the Michelson–Morley experiment and its outcome are described in the Appendix. Let me assume you have read it, and get on to the result of the experiment which was that there was *no evidence for an ether wind.* In other words, the velocity of light was the same, both in the direction of Earth's motion and in a direction perpendicular to it, or to put it differently, the velocity of light always remained the same, irrespective of whether the source or the observer moved. Contrast this with the case of sound.

This result was a big puzzle. If light has to propagate, then there must be the ether (at least so people thought then). If there is this ether, then there must be an ether wind when the Earth moves around the Sun. But even very careful experiments revealed no trace of this wind!

3.5 Lorentz contraction

Is there a way of understanding this **null** result, i.e., could one have the ether and yet no ether wind? One way of achieving this is by giving up the Galilean transformations because they predict that the velocity of light would change when the observer moves, which essentially means that there is an ether wind. Sorry, we can't give up the Galilean transformations because that would mean giving up the principle of relativity handed down by Newton. In other words, people at that time wanted the ether, *and* the Galilean transformations *but not* the ether wind. This is like having the cake and eating it too! And so, people came up with all sorts of clever ways of doing this. Let me now describe a few of these attempts.

In 1892, FitzGerald suggested that bodies are contracted in the direction of motion relative to the ether. According to this idea, the length of one arm of the Michelson interferometer is shrunk while the other is not; and the shrinking is by just the right amount to mask the presence of the wind—see Fig. 3.2. Lorentz in Holland (see Box 3.2) justified FitzGerald's idea on the basis of an electron theory of matter. This contraction in the direction of motion is now referred to as FitzGerald–Lorentz contraction or simply as *Lorentz contraction.* So, contraction is one way of having the ether, but still, explaining away the null result.

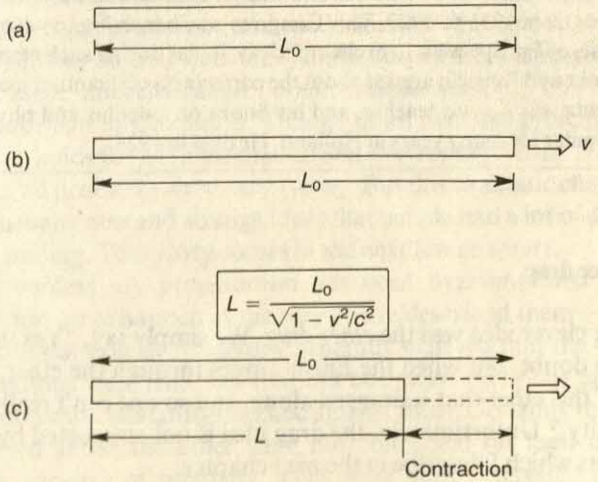


Fig. 3.2 This figure illustrates schematically the idea of Lorentz contraction. (a) shows a stationary rod of length L_0. In Newtonian relativity the length of the rod remains L_0, no matter what its velocity v is—see (b). When Michelson and Morley obtained their famous null result, FitzGerald in Ireland and Lorentz in Holland independently suggested that the null result was due to a contraction effect. According to them, a moving rod is contracted in the direction of its motion as in (c). This contraction, now known as Lorentz contraction, follows automatically from Einstein's special relativity.

Box 3.2 Hendrik Antoon Lorentz was born in Holland in 1853. He was very good at studies, always standing first. Mathematics was his favourite subject but he also liked literature and history. Besides, he was good at many European languages. This helped him later when he became an international figure.

While he worked as a school teacher he was also registered as a doctoral student. He got his doctorate degree at the age of 21 for his thesis in theoretical optics. For a brief period he was not certain whether he should pursue mathematics or physics; eventually he settled for mathematical physics.

Lorentz is best known for his theory of the electron and for the work which paved the way for Einstein's theory. In the second half of nineteenth century, many grappled with the problem of electricity and magnetism. Among them was Maxwell, but neither his papers nor his book made much sense to the physicists on the European continent. Lorentz was one of the early European physicists to understand the importance of Maxwell's work. Using his electron theory, Lorentz explained the Zeeman effect discovered by another Dutch physicist named Zeeman. For this, he shared the Nobel Prize with Zeeman in 1902.

For many years, Lorentz worked all by himself, avoiding contact with other physicists. In this respect, he was somewhat like Willard Gibbs. However, after 1900 he came out of seclusion and was the President of the famous Solvay Congress from 1911 to 1927. This Congress was a meeting to which leading physicists of Europe went to exchange ideas. It was during such meetings that Niels Bohr and Einstein argued about the correctness of quantum mechanics.

Lorentz was a good teacher, and his books on calculus and physics were very popular for many years in Holland. He died in 1928.

3.6 Ether drag

Another clever idea was the *ether drag*. We simply say, "Yes there is this ether no doubt, but when the Earth moves through the ether, there is a cloud of this ether that is dragged along, and so one can't really measure its velocity." Unfortunately, the drag idea is not supported by other experiments which I describe in the next chapter.

With Michelson's null result we are back to square one. Light definitely seemed to be travelling at the same speed c in all inertial frames, suggesting that, as far as light is concerned at least, formula (2.3) and therefore also the Galilean transformations do not work. And, according to certain other experiments (to be described in Chapter 4), if there was an ether, it certainly was not being dragged. People thus had a problem on their hands.

3.7 What Einstein did about the problem

There were some feeble attempts to tinker with Maxwell's equations for solving this difficulty, but this did not find much favour since Maxwell's equations had done a tremendous job of explaining so many things connected with electromagnetism.

Till the Michelson–Morley experiment, Galilean invariance ruled supreme and protected the principle of relativity (as enunciated by Newton). But it failed where electromagnetism was concerned, and if the principle of relativity had to be applied to electromagnetism then one simply had to give up the Galilean invariance as far as electromagnetism was concerned and substitute it with something else. But that was not satisfactory because we would then have one kind of protection device for mechanics and another for electromagnetism. By protection device I mean transformation equations which guarantee the principle of relativity. Could it be that the Galilean transformations were not OK? In that case, we would have to suspect many things that Newton wrote. But that is unthinkable since the ideas of Newton had been repeatedly demonstrated to be correct for nearly two centuries.

Interestingly, Albert Einstein did the unthinkable. He simply ignored the Galilean transformations and made suitable changes to the mechanics of Newton so that *both* mechanics and electromagnetism came under the same umbrella called *Lorentz transformations.* You will hear more about them in Chapter 5. Thanks to all this, the principle of relativity now holds for both mechanics *and* electromagnetism. Suddenly, like magic, all problems were swept away. But this dramatic change also brought in many new and strange ideas that people had a lot of difficulty in understanding. That story comes in the next few chapters.

I must confess my presentation has been oversimplified because things did not quite happen in the way I have described them—more or less, yes, but not exactly. Everyone generally believed that the Galilean transformations were true, but that did not mean every law and every phenomenon was constantly checked against them. Certainly the people who worried about the ether were not concerned (at least initially), about the principle of relativity. They were more interested in trying various experiments to get a feel for this mysterious ether. Maxwell also believed in the ether, and he never bothered to check what happened to his equations when subjected to a Galilean transformation.

The early experiments on the ether were rather crude and not definitive. In fact, Maxwell even wondered if more accurate experiments could ever be done. And then came Michelson who did what then seemed impossible. Once the significance of the null result obtained by him began to sink in, people began to panic, so to speak; hence ideas like contraction.

At this stage, it was discovered that Maxwell's equations were not covariant under Galilean transformations, and the roof seemed to come down. Luckily there were other transformations which did the job, and Einstein exploited them to discover the Special Theory. I shall tell you a bit more about the history and the personalities involved in Chapter 9. For the moment let us try to understand Einstein's theory.

Appendix to Chapter 3

A simplified version of the apparatus used in the Michelson–Morley experiment is shown in figure (a). Light from a source falls on a partially-silvered mirror M. Part of the incoming beam is transmitted, and this part travels upto the mirror M_1, gets reflected by M_1 and comes back to M. The second part of the incoming beam is reflected by M, travels to mirror M_2, gets reflected there, and returns to M. The two beams, i.e., those reflected by M_1 and M_2 now combine together and are viewed through the telescope T. As a result of interference between the two beams, what one sees in the telescope are interference fringes as shown in figure (b).

Let us now consider the time taken by the two beams 1 and 2 to travel from M upto M_1 and M_2 respectively and get back to M. Since there is an ether wind blowing in the direction M_1M with a velocity v, as per (2.3), light would have a speed $(c - v)$ during the forward journey and a speed $(c + v)$ during the return. For convenience, we shall call the two arms of the interferometer as A and B. The journey time t_A is therefore given by

$$t_A = \frac{l_1}{c - v} + \frac{l_1}{c + v} = \left(\frac{2l_1}{c}\right) \cdot \frac{1}{(1 - v^2/c^2)}.$$

Now consider beam 2. Once again we want the travel time for the journey MM_2M. In calculating this, we must be careful for, by the time beam 2 reaches M_2 after leaving M, M_2 would have moved away from its original position—see figure (c). Likewise, by the time beam 2 arrives back at M, the latter too would have moved—again see (c).

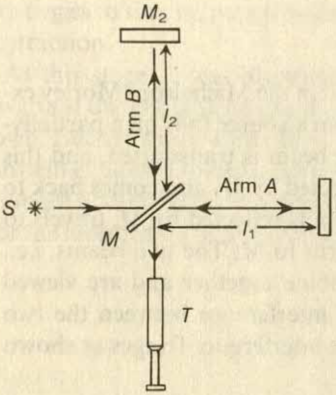
So, we must really calculate the time for travel along the path shown by the dashed lines. This is not very difficult, and one gets the travel time t_B as

$$t_B = \left(\frac{2l_2}{c}\right) \cdot \frac{1}{\sqrt{1 - v^2/c^2}}. \qquad \text{(\textit{Hint}: Use Pythogoras' theorem)}$$

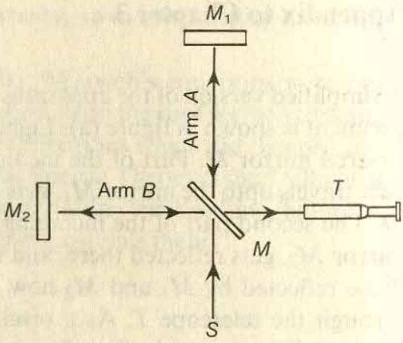
The difference in transit time is

$$\Delta t = t_B - t_A.$$

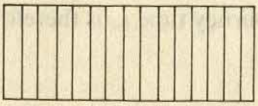
Michelson now rotated the apparatus by 90° as sketched in figure (d) M_1 and M_2 interchange places, and this time light moves with or against the ether wind along the arm with length l_2. Once again one can calculate the individual travel time t'_A and t'_B and the difference $\Delta t' = t'_B - t'_A$.



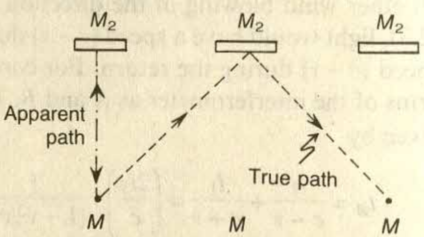
(a)



(d)



(b)



(c)

One can now calculate the difference

$$(\Delta t' - \Delta t) = \delta \text{ (say)}.$$

If λ is the wavelength of the light that is used, then the period of light vibration is λ/c. Therefore, when the apparatus is rotated, the fringes would shift by an amount $(\delta c/\lambda)$. In the experiment of Michelson and Morley $(l_1 + l_2)$ was 22 metres, λ was 5.5×10^{-7}m, and $(v/c) = 10^{-4}$. So the expected fringe shift was about 0.4 fringes. This, by the way, is the shift expected from the ether theory. Michelson and Morley observed a fringe shift of less than 0.01 fringe—in other words, practically no shift. So they obtained a NULL result.

4 *The Michelson–Morley Experiment*

4.1 Disturbing the ether

The Michelson–Morley experiment is of such importance that it is worth devoting a whole chapter to its history. Right from the time the wave theory of light was proposed, everyone imagined that light propagated in the ether. At the same time, people were also anxious to prove that the ether *did* exist. And so they began to think of various experiments to demonstrate the existence of the ether and the effects produced by it.

The Earth moves with a speed of about 30 km/sec, which is a pretty good speed indeed. It is not easy to produce that kind of speed in the lab. So people tried to see if the motion of the Earth through ether resulted in any visible effects. The experiments were all optical in nature, but in those days very high experimental accuracy was not possible. Most of these experiments therefore did not reveal anything definitive about the ether.

4.2 On the accuracies needed

While discussing such experiments, people usually talk of first-order experiments, second-order experiments, etc. Order in this case is characterised by the parameter (v/c). If we take v to be the velocity of the Earth, then (v/c) is about 10^{-4}. This is the accuracy of the first-order experiments. Second-order experiments would have an accuracy of about $(v/c)^2$, i.e., one part in a hundred million and so on.

Soon after he developed his electromagnetic theory, Maxwell wrote,

> If it were possible to determine the velocity of light by observing the time it takes to travel between one station and another on the Earth's surface, we might, by comparing the observed velocities in opposite directions, determine the velocity of the ether with respect to the terrestrial stations. All methods, however, by which it is practical to determine the velocity of light from terrestrial experiments depend on the measurement of the time required for the double journey from one station to the other and back again, and the increase of this time on account of the relative velocity of the ether equal to that of the Earth in its orbit would be only about one hundred millionth part of the whole time of transmission, and would therefore be quite insensible.

What Maxwell was saying is that light would travel at different speeds, depending on whether it was propagating "upstream" or "downstream". Unfortunately, all methods to determine the velocity of light involved sending a beam of light from a particular spot, and then making it come back to the same point (see Appendix 1). These methods did not allow the timings for the up- and the down-trips to be measured separately.

The beauty of experimental physics is that what seems impossible at one time, becomes possible sooner or later. And the man who made possible the experiment, which Maxwell said was near impossible, was Albert Michelson.

4.3 Aberration experiments

Before I come to the events leading to the famous Michelson–Morley experiment, I should say something about a few other experiments. The first of these is the *aberration experiment.* Say there is a star "vertically above" in the sky as schematically shown in Fig. 4.1(a). If we try to observe it through a telescope, as in the figure, we would not see the star! Why? Because, the Earth is moving. Suppose the motion with respect to the star is in the direction of the arrow in Fig. 4.1(b). Then the telescope must be tilted by an angle α with respect to what we thought should be

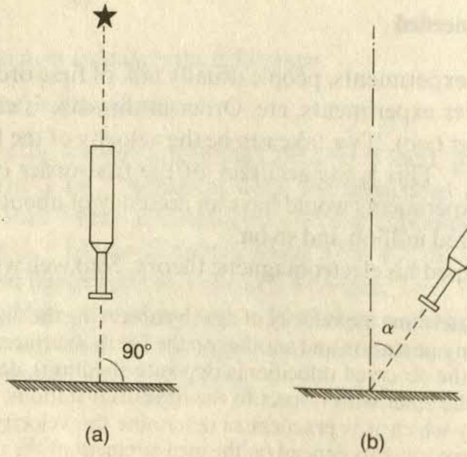


Fig. 4.1 (a) shows a star vertically above. If the Earth were not moving, we would have to orient the telescope as in (a) in order to see the star. However, if the Earth (and therefore also the telescope) were moving in the direction shown in (b), then the telescope would have to be tilted by an angle α in order to observe the star.

the direction of observation. This is similar to our tilting the umbrella when walking in the rain, even though the rain drops are falling vertically. The effect is called *aberration*, and the angle α is called the angle of aberration.

Figure 4.2 illustrates how to calculate α. Let Δt be the time taken for light to pass vertically down through the telescope. During this time, the light would have travelled a distance $c \cdot \Delta t$. In the same period, the telescope would have travelled a horizontal distance $v \cdot \Delta t$. So the angle of tilt of the telescope is given by

$$\tan \alpha = (v \cdot \Delta t)/(c \cdot \Delta t) = v/c. \tag{4.1}$$

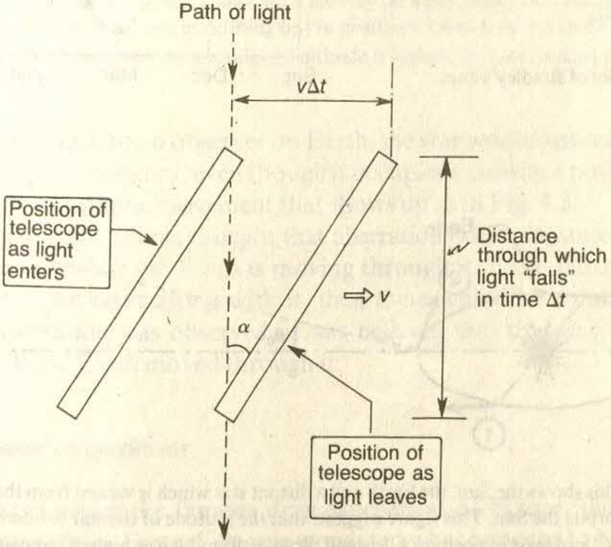

Path of light
$v\Delta t$
Position of telescope as light enters
Distance through which light "falls" in time Δt
v
α
Position of telescope as light leaves

Fig. 4.2 This figure illustrates how the aberration angle α of the previous figure can be estimated. By tilting the telescope, one enables the light to "fall" through the telescope unobstructed, while the telescope is in motion. Angle α is calculated as in equation (4.1).

Since (v/c) is about 10^{-4}, the angle α works out to be around 20 second of arc.

If the Earth moved on a straight line at a constant speed, we would never know that there was an aberration. But since it is going around the Sun, its velocity is changing and we are able to detect changes in the aberration.

In 1727, the astronomer Bradley made some interesting observations on the aberration of the star Draconius—see Fig. 4.3. To understand it, consider Fig. 4.4 which shows a star and four positions of the Earth as it

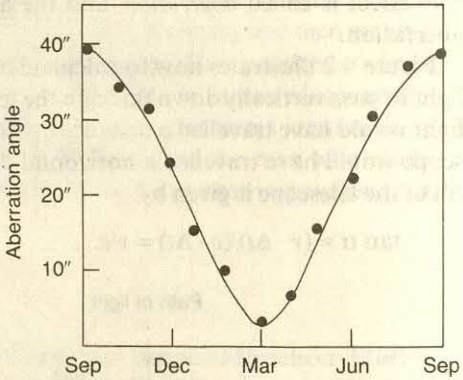


Fig. 4.3 Plot of Bradley's data.

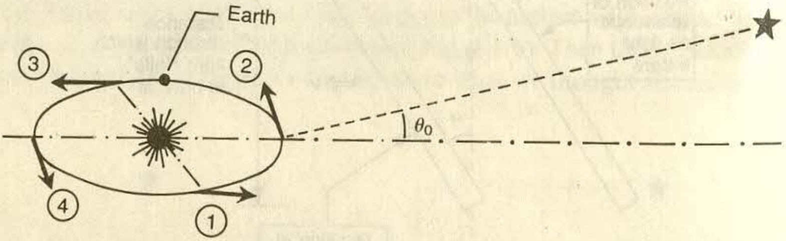


Fig. 4.4 This shows the Sun, the Earth and a distant star which is viewed from the Earth as the latter orbits the Sun. This figure suggests that the altitude of the star would be highest in position 2 and least in position 4. Instead, Bradley found it was highest in position 3 and least in position 1. This change is due to aberration.

goes around the Sun. From this it would seem that the altitude of the star would be greatest when the Earth is in position 2 and least in position 4. Instead, Bradley found it was highest in position 3 and least in position 1. All this is due to aberration.

Consider Fig. 4.5. The aberration is large when θ_0, the angle between the velocity vector and the line joining the star to the Earth, is large and is small when θ_0 is small. In positions 2 and 4, θ_0 is 90°; hence the magnitude of the aberration is largest, being equal to $\pm$ (v/c). By a similar argument, the magnitude of the aberration is the least in positions 1 and 3 (having the value $\pm$ ($v \sin \theta_0/c$). When the effects of aberration as noted above are taken into account, the altitude of the star gets modified as described

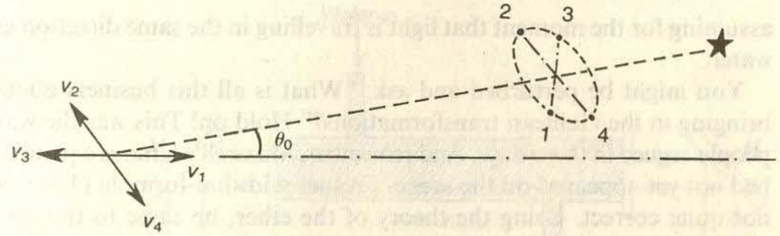


Fig. 4.5 The aberration effect depends on the angle θ_0 between the velocity vector and the line joining the star to the Sun. (Since stars are very far away, it does not matter if we take the Sun instead of the Earth in defining θ_0.) In positions 2 and 4, θ_0 equals 90°; therefore, aberration is a maximum. As a result, the altitude is highest at 3 and least at 1, exactly as Bradley found.

earlier. In short, to an observer on Earth, the star would appear to move on an elliptic trajectory, even though it occupies a constant position. It is this apparent elliptic movement that shows up as in Fig. 4.3.

In those days, people thought that aberration occurs because the ether remains still while the Earth is moving through it. If the Earth *dragged* or carried the ether along with it, then there would be *no* aberration. Since aberration was observed, it was believed that the *ether remained still* while the Earth moved through it.

4.4 Fizeau's experiment

We next consider the famous drag experiment of Fizeau, the idea for which came from Fresnel. People wondered what happens to light when it propagates through a medium which is itself moving. There must be ether inside the moving medium. Will this ether be stationary while the medium is moving, or will it be dragged along with the medium? How will all this affect the velocity of light?

Fresnel argued (on the basis of the ether theory of course) as follows: Let the velocity of light in vacuum be c. For the sake of argument, let us suppose the medium is water. To start with, let us assume that the water is stationary. One knows that the velocity of light in a medium is different from that in vacuum. In fact, it is given by

$$v_{\text{medium}} = c/n \tag{4.2}$$

where n is the refractive index. When the water moves, we apply the Galilean transformation and find

$$v_{\text{moving medium}} = (c/n) + \text{velocity of water}, \tag{4.3}$$

assuming for the moment that light is travelling in the same direction as water.

You might be perturbed and ask: "What is all this business about bringing in the Galilean transformations?" Hold on! This was the way people argued in those days. And remember, Maxwell's famous equations had not yet appeared on the scene. Fresnel said that formula (4.3) was not quite correct. Using the theory of the ether, he came to the conclusion that the velocity of light in the medium is given by the modified formula

$$v_{\text{moving medium}} = (c/n) + v_{\text{water}} \cdot f \tag{4.4}$$

where f is a fraction called the *Fresnel drag coefficient* whose value is

$$f = [1 - (1/n^2)]. \tag{4.5}$$

In deriving this formula, Fresnel made a number of assumptions which we today know to be incorrect. But remarkably, the formula itself is correct!

What were the assumptions Fresnel made? First, he accepted the basic validity of the Galilean transformations. Secondly, he said that there was ether in the water in addition to the ether that was there before the water was introduced. He assumed that only the ether in the water was dragged and *NOT* the ether that belonged to that region of space, i.e., he assumed *partial* drag. I don't know if you follow all this; I have my own difficulty! Anyway, Fresnel said that since there is only partial drag, the velocity of light in the moving medium is not quite given by Galileo's formula, i.e., (4.3). Rather, the water speed contributes only an amount $v_{\text{water}} \cdot f$.

In 1851, Fizeau, known for his various experiments to measure the velocity of light, decided to verify Fresnel's formula for the partial ether drag. His apparatus is shown in Fig. 4.6. Light derived from a source is split into two beams. These are then made to travel through two tubes in which water is flowing. In one of the beams the light always travels in the same direction as the water flow and in the other it travels against the flow. The two beams are then combined whereupon interference fringes are formed. While performing the experiment, one initially arranges the water to be stationary in the two tubes. The flow is then started, and this causes the fringes to shift. From the fringe shift, formula (4.5) is verified.

In Fizeau's experiment, the length of the tube was 1.5 m and the speed of water flow was 7 m/sec. A fringe shift of 0.23 of a fringe was observed from which a drag coefficient of 0.48 was estimated. As against this,

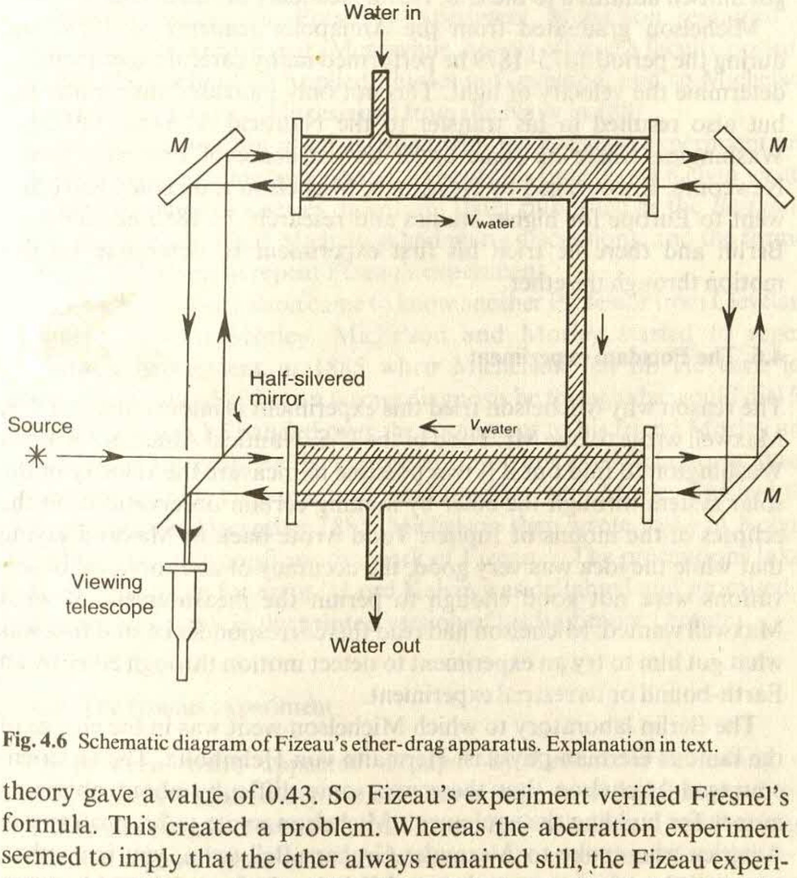


Fig. 4.6 Schematic diagram of Fizeau's ether-drag apparatus. Explanation in text.

theory gave a value of 0.43. So Fizeau's experiment verified Fresnel's formula. This created a problem. Whereas the aberration experiment seemed to imply that the ether always remained still, the Fizeau experiment seemed to suggest that the ether was being partially dragged. The Michelson–Morley experiment complicated matters even further because it suggested that the ether was fully dragged.

4.5 About Michelson

We now come to Michelson. Albert Abraham Michelson was born in Poland in December 1852. When he was four, his parents migrated to America. Albert's father worked as a shopkeeper in California. In school, Albert was attracted to science. Later, on the advice of the headmaster of his school, he appeared for a competitive examination to select cadets for the U.S. Navy. Three boys secured the first rank (!), Albert Michelson being one of them. With a letter from a local politician, young Albert went to America's capital city Washington, met the U.S. President and got himself admitted to the U.S. Naval Academy in Annapolis!

Michelson graduated from the Annapolis academy in 1873, and during the period 1875–1879 he performed many careful experiments to determine the velocity of light. This not only increased his reputation, but also resulted in his transfer to the Nautical Almanac Office in Washington where he came under the influence of Professor Simon Newcomb. Encouraged by Newcomb, Michelson took study leave and went to Europe for higher studies and research. In 1880 he landed in Berlin and there he tried his first experiment to determine Earth's motion through the ether.

4.6 The Potsdam experiment

The reason why Michelson tried this experiment is interesting. In 1879, Maxwell wrote to one Mr. Todd of the U.S. Nautical Almanac Office in Washington to find out if it was possible to measure the velocity of the solar system through the ether by making certain observations on the eclipses of the moons of Jupiter. Todd wrote back to Maxwell saying that while the idea was very good, the accuracy of astronomical observations were not good enough to permit the measurement of what Maxwell wanted. Michelson had read this correspondence, and that was what got him to try an experiment to detect motion through ether by an Earth-bound or terrestrial experiment.

The Berlin laboratory to which Michelson went was in the charge of the famous German physicist Hermann von Helmholtz. The latter encouraged Michelson, but there was some difficulty about obtaining money for building the equipment. Michelson wrote to his professor in America who spoke to Alexander Graham Bell (who, you remember, invented the telephone) and soon Michelson's financial problem was solved.

The experiment which Michelson did in Berlin is often referred to as the *Potsdam experiment* (Potsdam is a suburb of Berlin). It was the forerunner of the more famous Michelson–Morley experiment. After performing the experiment Michelson wrote, "The interpretation of these results is that there is no displacement of the interference bands. The result of the hypothesis of a stationary ether is thus shown to be incorrect." This is a strange result if one thinks about it for a minute. Why strange? Because this experiment showed that the ether, if it existed, was *NOT* stationary. The aberration experiment, on the other hand, showed that the ether *WAS* stationary. More about this conflict soon.

4.7 The Fizeau experiment again

After completing the Potsdam experiment Michelson remained in Europe for one more year. Meanwhile, he was offered a faculty position in the Case School of Applied Physics in Cleveland, and so Michelson returned to America. He resigned from the Navy in 1881.

Michelson was not at all satisfied with his Potsdam experiment and was very keen to improve upon it. About this time, Lord Kelvin visited America and gave a series of lectures (later published as the *Baltimore Lectures*). Kelvin and Michelson had many discussions, and the former urged Michelson to repeat Fizeau's experiment.

Meanwhile, Michelson came to know another Professor from Cleveland named Edward Morley. Michelson and Morley started to repeat Fizeau's experiment in 1885 when Michelson fell ill. He then left Cleveland, and thanks to a wrong diagnosis he thought he would die! So before leaving, he handed over the equipment to his friend Morley and asked him to complete the experiment. Fortunately Michelson's illness was not serious. He came back, and together with Morley finished the experiment in December 1885. Michelson then wrote to Lord Kelvin, "The result fully confirms the work of Fizeau ... The precautions taken leave little room for error." Lord Kelvin was delighted and promised to include this result in the printed version of his Baltimore Lectures.

4.8 The famous experiment

In between, there appeared a paper by Lorentz commenting on Michelson's Potsdam experiment. Lorentz pointed out that the fringe shift must be half of what was calculated by Michelson. Lord Rayleigh, by now a good friend of Michelson, drew the attention of the latter to the observation of Lorentz. In reply Michelson wrote, "I have never been fully satisfied with the results of my Potsdam experiment ..."

And so with Morley, he started a new experiment, to improve upon what he had done in Potsdam earlier. About this experiment, Morley wrote to his father,

> Michelson and I have begun a new experiment. It is to see if light travels with the same velocity in all directions. We have not got the apparatus done yet and shall not be likely to get done for a month or two. Then we shall have to make observations for a few minutes every month for a year ...

Morley then goes on to describe the new apparatus. The apparatus was set up in the basement of the laboratory building because here the temperature did not vary much on account of the massive stone walls.

The apparatus was mounted on a big slab of granite which rested on a circular wooden plate. This plate was fitted into a cast iron trough containing mercury. The iron trough was itself mounted on a brick pier resting on a special concrete base—see Fig. 4.7.

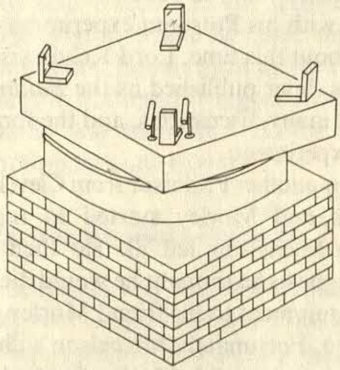


Fig. 4.7 Sketch of the apparatus used by Michelson and Morley in their famous experiment. What is shown is a rather simplified version. In reality, Michelson and Morley used a clever scheme with many mirrors which increased the path of light and therefore the accuracy of their experiment.

In October 1886, there was a fire which destroyed many things but fortunately the apparatus itself was saved by students from the neighbouring hostel. Finally, in 1887 Michelson and Morley were able to perform the experiment and report that instead of the expected fringe shift of 0.4, there was a shift of less than 0.01. Michelson then wrote to Lord Rayleigh, "The experiments on the relative motion of the Earth have been completed and the result decidedly negative."

Lord Rayleigh then promptly got in touch with Lorentz about Michelson's latest and much more accurate null result. Lorentz was completely puzzled. If one believed in the Fizeau experiment, then there was partial drag of the ether. The aberration experiment implied that the ether remained still. Either way, Michelson should *not* have obtained a null result. But the experiment *did* yield such a result, and now it seemed as if the ether was being *fully dragged*! Was there really an ether? If so, was it still, or partially dragged or fully dragged?

No wonder Michelson was completely surprised. As he wrote to Lord Rayleigh, "I am totally at a loss to clear away this contradiction." Lorentz then tried to clear up matters in his own way (via Lorentz contraction). He did not quite succeed in resolving all the difficulties, but the papers he wrote did have an influence over Einstein who "abolished" the ether and offered instead the Special Theory which at one stroke removed all the prevailing confusion. Thanks to this theory, *all* the earlier experiments could be interpreted *without* bringing in the ether. There was no conflict between them any longer.

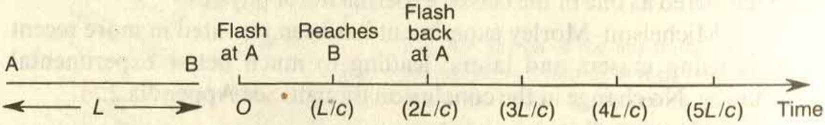
One last word about Michelson. He was the first American to receive the Nobel Prize. He will always be remembered as a first-class experimenter, not only because of the famous Michelson–Morley experiment, but on account of a number of other beautiful experiments as well.

As far as the famous null result of his is concerned, Einstein always maintained (as I shall describe later) that it (the null result that is) had no effect on him. Maybe Einstein had reasoned out the Special Theory of Relativity without being prodded by the result of Michelson and Morley. Nevertheless, their's was a landmark experiment which would always be remembered as one of the classic experiments of physics.

The Michelson–Morley experiment has been repeated in more recent times using masers and lasers, leading to much better experimental accuracy. No change in the conclusion though! See Appendix 2.

Appendix 1 to Chapter 4

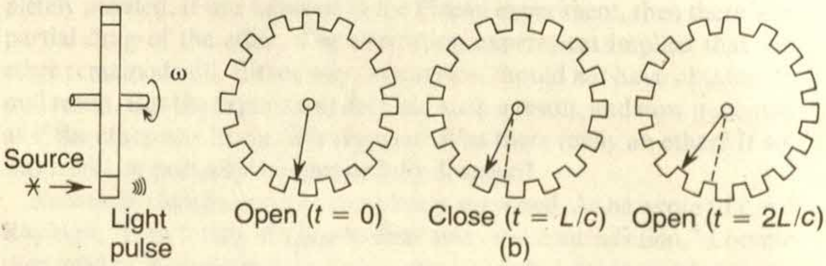
Consider two points A and B distance L apart—see figure (a). A pulse of light is emitted at A at some time which we call as zero time. The pulse travels to B where there is a mirror, gets reflected at the mirror and returns to A. If the total time for the round trip is t, then the velocity of light is simply $(2L/t)$.



(a)

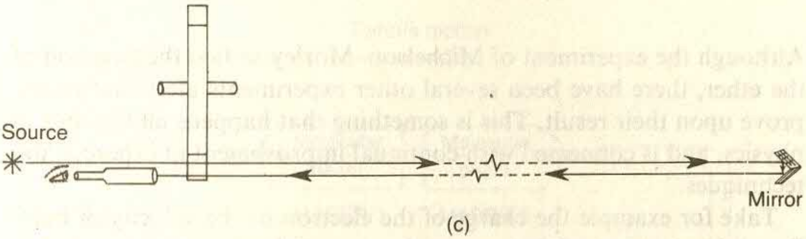
A number of measurements of the velocity of light were made during the last century, and in all these cases, the principle of measurement was just the above. However, no one ever measured the time t directly because such accurate clocks did not exist. For example, if L is 15 km and c is taken as 3×10^8 m/sec, then t is 0.0001 sec. This is indeed a very small time interval. How then did people manage to measure the velocity? That is where cleverness comes in!

Fizeau, the French physicist, used a cute method. He had a toothed wheel rotating very fast as in figure (b). The light beam is then "chopped" into flashes. Consider a flash. It travels from the wheel, gets reflected and travels back towards the wheel. When the flash or the light pulse reaches the wheel again, one of two things could happen. It faces a tooth in which case the pulse is blocked. Or it faces a gap in which case it emerges on the other side of the wheel—see figure (b). The way the experiment is done is as follows. One watches the wheel as in figure (c), and starts with a very slow speed. Since light travels very fast, at slow



Open ($t = 0$) Close ($t = L/c$) Open ($t = 2L/c$)

(b)



(c)

wheel-speeds the flashes would all get through and be visible to the eye. As the wheel speed is increased, a stage would be reached when the pulse on its return journey would encounter a tooth instead of a gap. Knowing the angle α between a gap and the adjacent tooth and also the angular velocity, one can calculate the time t required for the wheel to turn through the angle 2α. This is also the time for the light pulse to travel the distance $2L$. So, time t is measured in terms of the angular speed ω and 2α.

Appendix 2 to Chapter 4

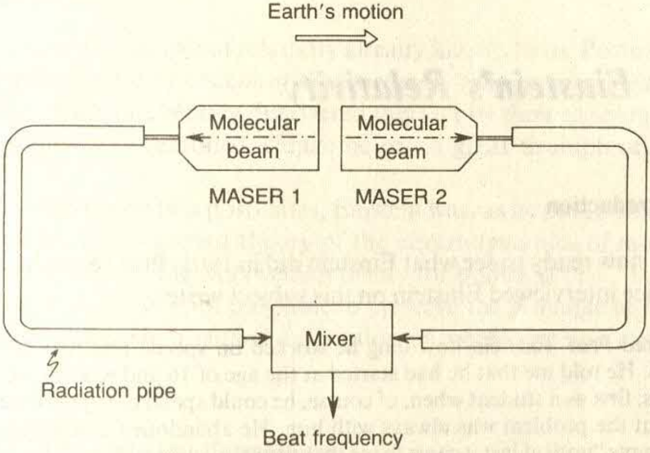
Although the experiment of Michelson–Morley settled the question of the ether, there have been several other experiments after that to improve upon their result. This is something that happens all the time in physics, and is connected with continual improvement in experimental techniques.

Take for example the charge of the electron or the velocity of light. People have measured these quantities accurately a long time ago, but as techniques improved physicists have tried to improve upon the accuracy of the earlier measurements.

In the case of the Michelson–Morley experiment, you will recall that they declared a null result because the measured fringe shift was about (1/50) of what was expected on the basis of the ether theory. In 1930, Joos in Germany showed that the fringe shift was roughly (1/500) of that expected if there was an ether. Don't jump to the conclusion that people were having doubts about the ether. Rather, it is the experimentalist's way of cautiously saying: "I certainly don't believe in the ether. However, as an experimentalist all I can say from experiments is that if there is ether, its effect is less than (1/100) or (1/500) compared to what the effect should be. That is all my experiment really allows me to say."

In 1958, Townes and his colleagues carried out an experiment which improved the accuracy to about (1/1000). Townes invented the MASER and also discovered the principle of the LASER for which he later received the Nobel Prize. For our purposes, the MASER is a microwave oscillator which emits microwaves of a fixed frequency. The way the maser radiation comes about is a different story and unfortunately I cannot go into it here. Let us just say that the maser is a highly monochromatic source of microwave radiation, and that the radiation comes from a molecular beam in the device.

Let ν be the frequency of the maser radiation. The value of ν depends essentially on the molecule used in the device. What Townes said was that if the maser is stationary in the ether, its frequency is ν. However, if the maser is moving with a velocity v with respect to the ether, then the frequency is shifted by a small amount $\Delta\nu$. Whether $\Delta\nu$ is positive or negative depends on how the molecules in the maser move with respect to the Earth's motion through the ether. Townes used two masers whose beams travelled in opposite directions as in figure (a). Therefore the frequencies of the two masers would be shifted as shown, provided there is relative motion with respect to the ether. Townes now mixed the two maser outputs giving a beat frequency of $2\Delta\nu$. (I hope you know what is



meant by beat frequency; or else look up any elementary textbook on sound). Townes then said that if the apparatus is rotated by 180°, there would be a net shift of 4Δν. Using the ether theory, he calculated

$$4\Delta\nu = \frac{4uv}{c^2} \cdot \nu \, .$$

Taking *v* to be the Earth's orbital velocity, he calculated that 4Δν should be ~ 20 Hz. His experiment showed that it was only ~ 1 Hz. From this he estimated that if *v* really denotes the velocity of the Earth through the ether, then it is only 30 m/sec instead of 30,000 m/sec. In other words, the accuracy of this null experiment is ~ (1/1000).

5 *Einstein's Relativity*

5.1 Introduction

We are now ready to see what Einstein did in 1905. Professor Shankland who once interviewed Einstein on this subject writes:

> I asked Prof. Einstein how long he worked on special relativity before 1905. He told me that he had started at the age of 16 and worked for ten years; first as a student when, of course, he could spend only part time on it, but the problem was always with him. He abandoned many fruitless attempts "until at last it came to me that time was suspect."

What Einstein means is that he began to wonder *whether clocks might behave differently while moving*; and that precisely is where all the trouble starts or the fun begins, depending on your point of view.

Curiously, Einstein was not bothered one bit about the null result of the Michelson–Morley experiment. But all the time he was thinking about light and realised that there was a clash between Newtonian relativity and electromagnetism. The mechanics of Newton respected the principle of relativity via the Galilean transformations. On the other hand, Maxwell's equations were a tremendous success and one had to believe in them, just as one placed great faith in Newtonian mechanics. However, Maxwell's equations definitely were *not* covariant under the Galilean transformations. Were there some other transformations which left these famous equations of Maxwell unchanged in form? If there were, how did such transformations affect Newtonian mechanics? It was these questions that Einstein raised and studied in his 1905 paper *On the Electrodynamics of Moving Bodies*.

5.2 The postulates of Einstein's theory

In his work, Einstein made two very important assumptions which are often referred to as the two *postulates* of the Special Theory of Relativity. They are:

1. The laws of physics are the same in all inertial frames, that is, no preferred frame exists.

2. The speed of light in free space has the same value c in all inertial systems.

Postulate 1 is the principle of relativity already known to us. Postulate 2 is sometimes called the *principle of the constancy of the speed of light*. No doubt Michelson and Morley discovered this fact by their experiments. But Einstein simply reasoned it must be so—a great triumph of pure thought!

Based on the above two postulates, Einstein was, as he put it, able "to give a simple and consistent theory of the electrodynamics of moving bodies on the basis of the Maxwellian theory for bodies at rest." In one sweeping stroke, he was not only able to preserve the principle of relativity, but also provide a *unified* description of mechanics and electrodynamics.

5.3 Enter the Lorentz transformations

Einstein's trick was to get rid of the Galilean transformations (which were giving trouble anyway) and replace them with something better; these are the *Lorentz transformations*. They are like the Galilean transformations in that they relate the space and time coordinates of inertial frames moving uniformly with respect to each other. An important difference between the two is that the Lorentz transformations *ensure the speed of light is the same in all the inertial frames* while Galilean transformations do not. As you might remember, according to the Galilean transformations, an observer towards whom the light source is travelling would measure for light a velocity greater than c (recall equation (2.3)), in contradiction to the second postulate of Einstein. That makes all the difference.

5.4 How to arrive at the Lorentz transformations

The *derivation* of Lorentz transformations involves some algebra and I shall not go into those details; but the basic idea is quite easy to understand. As before, we consider two frames S and S' moving uniformly with respect to each other with a velocity v. Space and time coordinates in S are denoted by (x, y, z, t) while those in S' are denoted by (x', y', z', t'). The coordinate x' depends on x, y, z, and t. Similarly for y' and z'. What about t'? That also depends on x, y, z, and t. Instead of clumsily using words let us write an equation, i.e.,

$$x' = Ax + By + Cz + Dt. \tag{5.1}$$

Here A, B, C and D denote some coefficients. For a reason which would become clear shortly, I shall instead write them as a_{11}, a_{12}, a_{13} and a_{14}. We thus have

$$x' = a_{11} \cdot x + a_{12} \cdot y + a_{13} \cdot z + a_{14} \cdot t \tag{5.2a}$$

and similarly,

$$y' = a_{21} \cdot x + a_{22} \cdot y + a_{23} \cdot z + a_{24} \cdot t \tag{5.2b}$$

$$z' = a_{31} \cdot x + a_{32} \cdot y + a_{33} \cdot z + a_{34} \cdot t \tag{5.2c}$$

$$t' = a_{41} \cdot x + a_{42} \cdot y + a_{43} \cdot z + a_{44} \cdot t. \tag{5.2d}$$

People who know about matrices would write the above equations compactly as

$$\begin{bmatrix} x' \\ y' \\ z' \\ t' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}. \tag{5.3}$$

It does not matter if you cannot follow the details. The point simply is that (5.3) defines some general relations between the primed and the unprimed coordinates. The only specific thing about these equations so far is that they define *linear* transformations, meaning that no powers higher than one occur, i.e., terms like x^2, x^3, z^4, t^2, etc., are absent on the right-hand side of (5.2).

So far, everything is quite general, and no reference has been made either to Galilean or Lorentz transformations. Everything now depends on the matrix of coefficients. If the matrix in (5.3) is given by

$$\mathbf{a} = \begin{bmatrix} 1 & 0 & 0 & -v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{5.4}$$

then the transformation equations (5.2) or (5.3) represent Galilean transformations (for S' moving with a velocity v with respect to S along the x-direction). What we now want is to choose the coefficients a_{11} etc., so that they respect the two postulates of relativity. This exercise is not very difficult but somewhat beyond our present scope. The final answer is:

$$\mathbf{a} = \begin{bmatrix} \gamma & 0 & 0 & -\gamma v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma v/c^2 & 0 & 0 & \gamma \end{bmatrix} \tag{5.5}$$

with $\gamma = 1/\sqrt{(1 - v^2/c^2)}$. From this we then have:

$$
\begin{aligned}
x' &= (x - vt)/\sqrt{1 - (v^2/c^2)} \\
y' &= y \\
z' &= z \\
t' &= \{t - (v/c^2)x\}/\sqrt{1 - (v^2/c^2)}.
\end{aligned}
\tag{5.6}
$$

These are the famous Lorentz transformations.

5.5 Lorentz transformations—some features

Comparing (5.6) with Galilean transformations, many striking differences become evident. The most important of these is that t' is no longer equal to t, which means that there is no such thing as a universal clock. On the contrary, *moving clocks run differently compared to stationary clocks*. What is even more striking is that t' depends on x, y and z which means that the way a moving clock runs would also depend on where it is! Peculiar things start happening as a result of all this, as I shall explain shortly.

The Lorentz transformations mix up space and time in an unusual way, and this was the source of all the confusion in the early days of special relativity. Figure 5.1 gives a qualitative picture of the transformations. I have been a bit sloppy in drawing this figure, but I shall soon make up for that. Here I have considered one space- plus one time-dimension, i.e., two-dimensional space-time. Suppose S and S' are two inertial frames, with S' moving uniformly with a velocity v with respect to S—see Fig. 5.1(a). The x–t axes in Fig. 5.1(b) look normal but the x'–t' axes are somewhat "skewed". The angle of tilt α depends on the velocity v.

5.6 Event, world and all that

The famous mathematician Minkowski refers to space-time as the *world* with a point in space-time such as P denoting an *event*. *The world is thus the totality of all possible events.* As already remarked, the motion of the particle is a succession of events, the line joining all these events being the *world line* of the particle.

The Lorentz transformations describe the relationship (in this case) between (x, t) on the one hand and (x', t') on the other. The $x' - t'$ axes appear "skewed" because the (Lorentz) transformations are not *orthogonal*. The two axes come closer and closer to each other as v increases, and when v becomes equal to c they coincide, giving a line inclined at 45° to the x-axis. This is the world line for a particle travelling

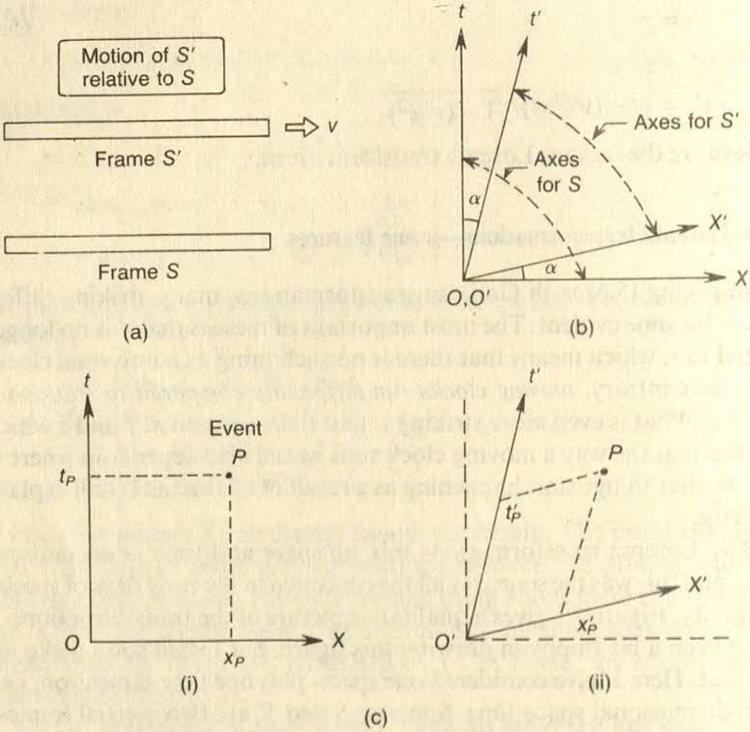


Fig. 5.1 S and S' are two inertial frames of which S' moves uniformly with a velocity v with respect to S as in (a). The axes of the two frames then appear as in (b). Notice the skew. (c) shows how the coordinates of an event P are obtained in the two frames.

at the speed c. One particle which travels at the speed c is the *photon* or the light quantum. Thus the world line of the photon is a line at 45°.

5.7 Measuring time in metres

While discussing Fig. 5.1 I had said I was being a bit sloppy. Remember? Let me explain what I meant. Firstly, in drawing space-time diagrams, one does not use t but rather $w = ct$. So, read w $(= ct)$ and w' $(= ct')$ in place of t and t', respectively.

In Fig. 5.1, I did not say anything about scales. I mean, suppose 1 cm along the x-axis represents 10 metres. Does 1 cm along the x'-axis also represent 10 metres? If not, how much does it represent? We shall now consider such questions, as also that of calibrating the x'- and the w'-axes.

Since $w = ct$, it has the dimension of length. Therefore, the w and the w' axes do not quite represent time but time disguised in terms of distance. In other words, while 1 metre along the x-axis *really means* 1 metre of distance, 1 metre along the w-axis does not represent 1 metre of distance but *the time taken by light to travel 1 metre of distance.* While all this might sound confusing, the advantage of using w and w' in place of t and t' is that the Lorentz transformations then take the form

$$\begin{aligned} x' &= (x - \beta w)/\sqrt{1-\beta^2} \quad & x &= (x' + \beta w')/\sqrt{1-\beta^2} \\ w' &= (w - \beta x)/\sqrt{1-\beta^2} \quad & w &= (w' + \beta x')/\sqrt{1-\beta^2} \end{aligned} \tag{5.7}$$

where $\beta = (v/c)$. Notice that the equations now look *symmetric* compared to (5.6); well, that is the advantage. Physicists often manipulate equations to make them appear symmetric, that is if they can.

5.8 A calibration exercise

Thanks to (5.7), we now have a method of calibrating the x'- and the w'-axes. Figure 5.2 explains how. Here besides the x- and w-axes, we also have the x'- and w'-axes. The angle ϕ is given by $\tan\phi = \beta$ or $\phi = \tan^{-1}\beta$. The maximum value β can have is 1, which happens when $v = c$. Since v is usually less than c, ϕ will be less than 45°.

Figure 5.2 shows many curves called *hyperbolae* (see also Box 5.1). Just as a circle can be described by the equation $x^2 + y^2 = a^2$, the hyperbola can be described by the equation $x^2 - y^2 = a^2$ (a = constant). The hyperbolae in Fig. 5.2 are described by:

$$x^2 - w^2 = 1 \qquad \text{(curve } A\text{)} \tag{5.8a}$$

$$x^2 - w^2 = 2 \qquad \text{(curve } B\text{)} \tag{5.8b}$$

$$x^2 - w^2 = 3 \qquad \text{(curve } C\text{)} \tag{5.8c}$$

Observe that corresponding to each of the equations above, there are actually two branches to the hyperbola.

Consider now the point x_1. The distance Ox_1 would represent unit distance along the X-axis—this is easily seen by putting $w = 0$ in equation (5.8a). This "unit" distance could represent 1 cm, or 1 m, or 1 km; that depends on the problem, convenience of representation and so on. Now consider the distance Ox'_1; it represents the same as what Ox_1 does. As you can easily see from the figure, the physical length of Ox'_1 is greater than that of Ox_1. So the X'-axis is a bit "stretched" or "dilated" compared to the X-axis; the higher the velocity v, the greater is this stretching—see Fig. 5.3.

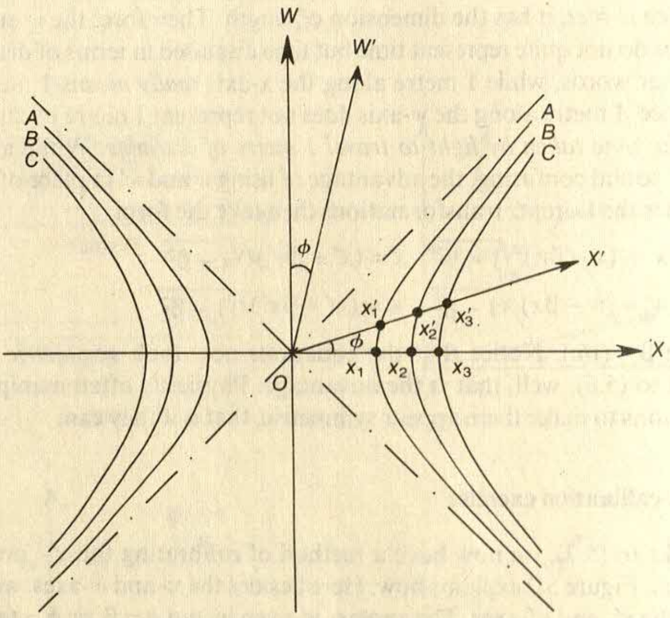


Fig. 5.2 This figure illustrates how the OX and the OX' axes are calibrated. First, one draws the various hyperbolae given in equation (5.8). If Ox_1, Ox_2 and Ox_3 represent, say, 1, 2 and 3 metres respectively along OX then Ox'_1, Ox'_2, and Ox'_3 represent 1, 2 and 3 metres along OX'. Observe that Ox'_1 is greater than Ox_1, which means that the X'-axis is "stretched" with respect to the X-axis.

The calibration of the W'-axis is done similarly, and is illustrated in Fig. 5.4. In this case the equation for the hyperbolae are given by $w^2 - x^2 = \text{constant}$ and this time it is the W'-axis which is "stretched" (compared to the W-axis).

5.9 Clocks in relativity

Clocks play an important role in special relativity. Obviously they are used for measuring time, and if we want to know whether two events occurring in two different places are simultaneous or not then there must be clocks in the two places. Naturally, they must also be *synchronized.*

Consider an example. To start with we assume that everyone adjusts his clock or wristwatch using the radio time-signal. Let us say there is a police officer who is questioning a suspect. "Where were you when the murder was committed?" he asks, and the suspect replies, "I was in the Udipi Hotel having coffee." The police officer then cross checks by

Box 5.1 We dig a little deeper into the hyperbola business. Some of the material presented here would perhaps make better sense after you have read section 5.16. So maybe you could go through this Box a second time after you have read that section.

In trigonometry, there is the famous result

$$\cos^2\theta + \sin^2\theta = 1. \qquad (1)$$

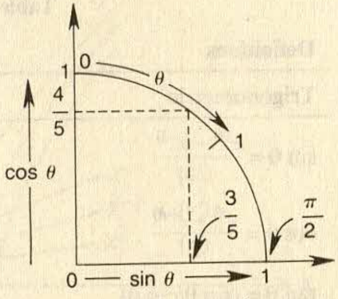
If I multiply throughout by r^2, I get

$$r^2\cos^2\theta + r^2\sin^2\theta = r^2. \quad (2)$$

Now look at figure (a). We see that $x = r\cos\theta$ and $y = r\sin\theta$. Therefore, (2) can be rewritten

$$x^2 + y^2 = r^2 \qquad (3)$$

which of course is the equation of the circle.



$(3/5)^2 + (4/5)^2 = 1$

(a)

What I have said so far is undoubtedly very familiar to you already. Now let us consider *hyperbolic* sines and cosines. These are written as sinh θ and cosh θ respectively, and are defined as

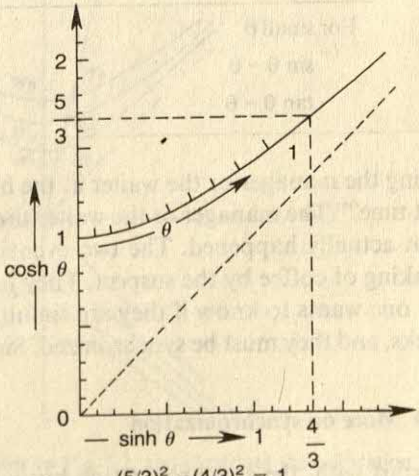
$$\sinh\theta = -i\sin(i\theta)$$

$$\cosh\theta = \cos(i\theta) \qquad (4)$$

where $i^2 = -1$. Let us now use (4) in (1). We then get

$$\cosh^2\theta - \sinh^2\theta = 1 \qquad (5)$$

which is the hyperbolic equation corresponding to the standard trigonometric equation (1). From (5) we can write



$(5/3)^2 - (4/3)^2 = 1$

(b)

$$a^2\cosh^2\theta - a^2\sinh^2\theta = a^2 = \text{constant} \qquad (6)$$

and the hyperbola is a graphical plot of the above equation—see figure (b). Table 1 gives you a quick summary of some of the properties of circular (i.e., trigonometric) and hyperbolic functions.

Table 1

Definitions

Trigonometric	Hyperbolic
$\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$	$\sinh\theta = \dfrac{e^{\theta} - e^{-\theta}}{2}$
$\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$	$\cosh\theta = \dfrac{e^{\theta} + e^{-\theta}}{2}$
$\tan\theta = (\sin\theta/\cos\theta)$	$\tanh\theta = (\sinh\theta/\cosh\theta)$

Relations

Trigonometric	Hyperbolic
$\sin(-\theta) = -\sin\theta$	$\sinh(-\theta) = -\sinh\theta$
$\cos(-\theta) = \cos\theta$	$\cosh(-\theta) = \cosh\theta$
$\cos^2\theta + \sin^2\theta = 1$	$\cosh^2\theta - \sinh^2\theta = 1$
For small θ	
$\sin\theta \sim \theta$	$\sinh\theta \sim \theta$
$\tan\theta \sim \theta$	$\tanh\theta \sim \theta$

asking the manager or the waiter in the hotel: "Was the suspect here at that time?" The manager or the waiter then say yes or no, depending on what actually happened. The two events here are the murder and the drinking of coffee by the suspect. They happen at two different places, and one wants to know if they are simultaneous. So there must be two clocks, and they must be synchronized. Simple enough, isn't it?

5.10 More on synchronization

To start with, the synchronization business appears quite simple. As I just told you, we normally set our clocks by using the time signal broadcast on the radio. Einstein recommends more or less the same procedure. Consider first an inertial frame *S*, say. There are fellows sitting every hundred metres or so, and each has a clock—a stopwatch is better—all identical and perfect. At some instant, the fellow at the origin sends a

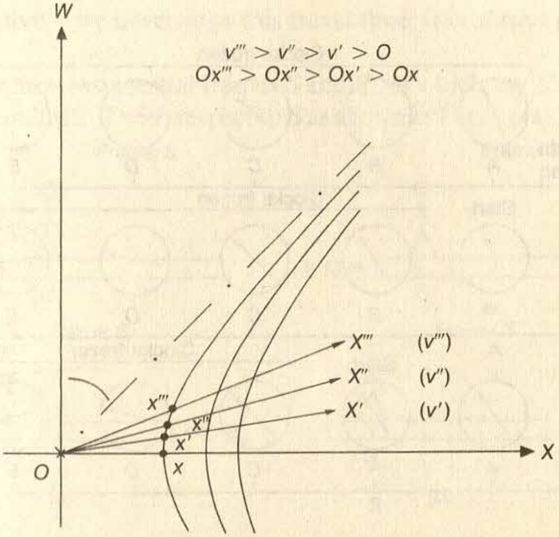


Fig. 5.3 This figure illustrates how the tilt of the space axis varies with velocity. Clearly Ox''' is greater than Ox'', which is greater than Ox', which in turn is greater than Ox.

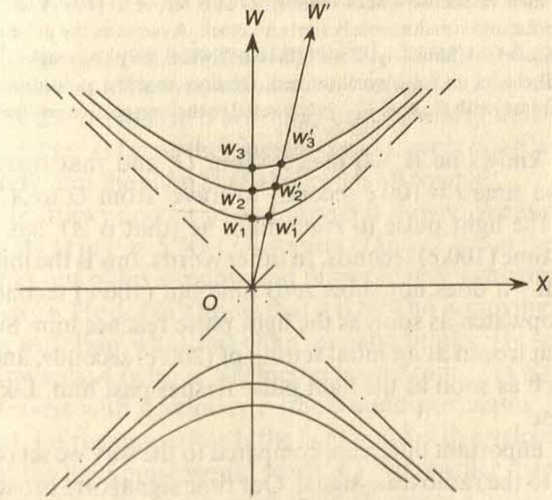


Fig. 5.4 This figure is similar in spirit to Fig. 5.2, and illustrates the calibration of the W'-axis. If Ow_1 represents 1 unit along W-axis (e.g., 1 metre of time), then Ow_1' represents the same unit (i.e., 1 metre of time) along OW'.

light flash. Simultaneously, he also starts his watch which initially was stopped and was showing zero time. The light pulse propagates and as soon as it reaches *A* (see Fig. 5.5), the latter starts his clock. But *A* is

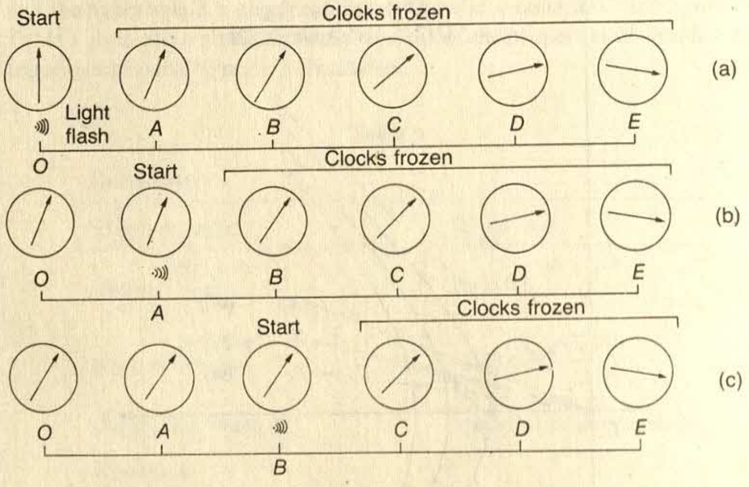


Fig. 5.5 Synchronization of clocks in relativity. (a) shows identical clocks, one every 100 metres (say) in a frame *S*, in charge of persons *O*, *A*, *B*, *C*, ... etc. To start with, all these persons freeze their respective clocks suitably, *O* at 0 sec, *A* at (100/*c*) sec., etc. *O* then flashes a light pulse and simultaneously starts his clock. As soon as the pulse reaches *A*, he starts his clock, as in (b). Similarly, *B* starts his clock when the pulse reaches him—see (c). In this way, all clocks in *S* are synchronized. Observe that the procedure described is *internal* to the frame, with *O*, *A*, *B*, *C*, ... etc., rooted to their respective positions.

careful. He knows he is 100 meters from *O*, and that therefore light would take a time $t = 100/c$ seconds to travel from *O* to *A*. So, while waiting for the light pulse to reach him, he (that is *A*), has frozen his clock at the time $(100/c)$ seconds. In other words, this is the initial setting of *A*'s watch—it does not show zero time but $(100/c)$ seconds. And *A* starts his stopwatch as soon as the light pulse reaches him. Similarly, *B* has his watch frozen at an intial setting of $(200/c)$ seconds, and he starts his stopwatch as soon as the light pulse flashes past him. Likewise also *C*, *D*, *E*, ... etc.

Notice an important difference compared to the way we set our watches by listening to the radio time-signal. Our time signals are broadcast from Delhi, but a person in Trivandrum (Thiruvananthapuram these days!) setting his watch does not allow for the time taken by the radio signal to travel from Delhi to Trivandrum. This is OK because the time required for travel is negligibly small and most of our watches are not that accurate. But if there is an astronaut on the Moon who wants to set his watch by tuning in to a radio station on Earth, then the time taken for the signal to travel from the Earth to the Moon becomes significant. In special relativity, we never miss this travel time; it is *always* taken into account.

Consider now two inertial frames S and S', of which say S' is moving to the right uniformly with respect to S as shown in Fig. 5.6(a). There are

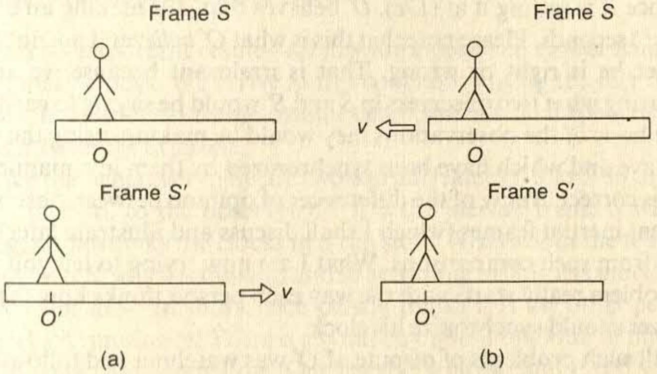


Fig. 5.6 S and S' are two frames of which say S is fixed while S' is moving uniformly as in (a). O and O' are observers in the two frames, located at the respective origins. The point of view of O' is shown in (b). He thinks his frame is stationary, and it is S which is in uniform motion. How he views the synchronization in S is described in the text.

clocks in S, and they are being synchronized. A chap in S' sitting at the origin O' is watching the synchronization business as it is being done (in S that is). All the persons in S are doing exactly what I described a little while ago, and so, as far as they are concerned, everything is being done quite properly. But the chap at O' thinks it is all wrong!

This is a bit tricky and let us go through it slowly. We start when the two frames are as in Fig. 5.6(a). Observers O and O' are at the same position. O' looks out of his window and thinks that S is moving to the *left* with a velocity v. He doesn't know that it is he who is moving, but we do; anyway, let us follow what this chap sees and thinks. He sees the light travelling towards A (who is 100 metres away from O in S). He knows that light travels with a velocity c (the second postulate); and he also believes that A is rushing towards the light pulse with a velocity v. So it is natural that the light pulse would reach A a little earlier than it would if A were not moving. Being a careful chap, O' makes a little estimate. He says: "Well if A were not moving, light would take a time (L/c) to go from O to A (where the distance OA equals L). But A is moving towards the light source, and so the light pulse would actually take less time; whose value is approximately $[L/(c-v)]$." This argument is easy to follow, and the difference between the time estimated by O and by O' is

$$(L/c) - \{L/(c-v)\} = (L/c)\cdot[1 - \{c/(c-v)\}] \sim -(Lv/c^2). \qquad (5.9)$$

Don't get scared by the negative sign. It simply means that the light signal arrives early (according to O' that is), compared to when it would if the light pulse and A were not rushing towards each other. So, according to O', the clock at A must be set not to (L/c) but to $[L/(c - v)]$; but since A is setting it at (L/c), O' believes that A is making an error of $-(Lv/c^2)$ seconds. Please note that this is what O' *believes*. I am not saying whether he is right or wrong. That is irrelevant because we are just comparing what two observers in S and S' would be saying to each other on the basis of the observations they would be making, using the clocks they have and which have been synchronized by them in a manner they think is correct. Many of the differences of opinion between observers in different inertial frames (which I shall discuss and illustrate later) come about from such comparisons. What I am now trying to tell you is that the problem really starts with the way each person thinks how the other observer should synchronize his clock.

In all such problems of dispute, if O' was watching and following the synchronization, he would say the synchronization itself was wrong and blame everything on that. But if he was ignorant about the synchronization, he would simply say that O is wrong, and that clocks in S *run slow*.

Suppose now that clocks in S' are being synchronized, and that a observer in S is watching the proceedings. He too would complain that the chaps in S' who are synchronizing the clocks are doing it all wrong! One of the funny things about relativity is that in many such arguments, both the parties may be right, even though they are fighting amongst themselves about who is right and who is wrong! You wonder: "How can that be?" Well, wait and see!

5.11 Is a perfect clock possible?

Is it at all possible to invent a clock which would never run slow while moving? This is simply *NOT* possible. The key point is that an *observer in an inertial frame cannot determine his own speed* without any reference to things external to his frame. Let us now hear what Feynman has to say about this issue. He says: First imagine that O gives to O' a super-duper clock which simply does not run slow. O' now says goodbye to his friend O, climbs into a spaceship and pushes off at a speed close to c. On board the spaceship, O' has this fancy clock along with all his usual types of clocks. Over now to Feynman:

> One of these [super-duper] clocks is taken into the space ship [by O'], along with the first kind. Perhaps *this* [super-duper] clock will not run slower, but will continue to keep the same time as its stationary counterpart, and thus disagree with the other moving clock. Ah no, if that should happen, the

> man in the ship could use this mismatch between his two clocks to determine the speed of his ship, which we have been supposing is impossible. *We need not know anything about the machinery* of the new clock that might cause the effect—we simply know that whatever the reason, it will appear to run slow, just like the first one.

This is an important result—*all* moving clocks run slower compared to stationary clocks. We arrive at this conclusion using the fact that an observer in an inertial frame cannot determine his own speed without reference to things external.

Notice that wherever there are two inertial frames S and S' in uniform relative motion, to the observer O', it is the inertial frame S which is moving; so, he thinks the clocks in S run slow. What about the fellow O? He thinks it is S' which is moving, and so according to him, it is the clocks in S' which run slow. In short, each person thinks it is the other person's clocks which run slower! There is a symmetry about this kind of thinking.

All moving clocks run slow. Our heart is a clock, and it too will run slow if we travel with speeds close to c! In fact, this is the basis for the famous twin paradox which I shall discuss later.

5.12 Space contraction

We now consider an important effect of special relativity which caused much confusion in the early days. I am referring to *space contraction*.

Let us say there are two metre-sticks, one in the inertial frame S and another in the inertial frame S' which is moving uniformly with a velocity v with respect to S. A chap in S' is trying to measure the length of the stick in S. You might wonder what there is to measure. For isn't a metre-stick always 1 metre long? Yes, but this fellow in S' is a suspicious character and wants to check things for himself. When he does this measurement, lo and behold, he finds that the metre-stick in S is in fact *less* than 1 metre long!

How can that be? Figure 5.7 offers the explanation. Now before I get on to this figure, let us forget relativity for a moment and simply think about how one would measure the length of a moving object. Say that the object is a fish in a tank, as illustrated in Fig. 5.8. For our convenience, the tank has a scale attached to it. We cannot measure the length of the fish by first marking the position of its tail, and then running ahead to mark the position of its head. This will not do because by the time we move over from the tail to the head, the fish itself would have moved as shown in Fig. 5.8(a). Obviously we must mark the position of the head and the tail *at the same instant*—see Fig. 5.8(b).

Let us now get back to Fig. 5.7. Here A and B are the world lines of the

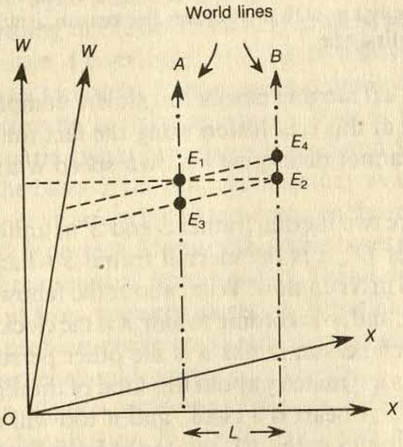


Fig. 5.7 Measuring the length of a stick in two frames S and S'. Let us say the stick is in S and that S' is moving uniformly with respect to S. The (X, W) and (X', W') axes would then be as shown. A and B are world lines of the stick in S. The distance $L = E_2E_1$ is the length of the stick in S. In S', the length is given by E_3E_2 (= E_1E_4).

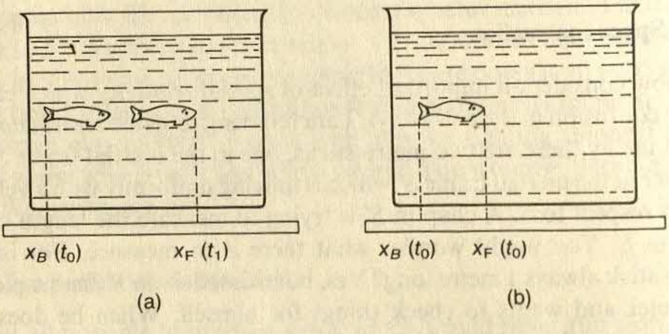


Fig. 5.8 Two ways of measuring the length of a fish swimming towards the right at a steady speed. In (a), we first mark the position x_B of the tail, run around to the front, and mark the position x_F of the head. Obviously, during this period, the fish would have moved. So $[x_F(t_1) - x_B(t_0)]$ cannot be the true length of the fish. To obtain the length, the head and the tail must be marked simultaneously as in (b).

two ends of a metre-stick that is stationary in S. E_1 and E_2 are the two events which correspond to observations made on the two ends of the stick. Obviously, they are simultaneous in the frame S and so the distance E_1E_2 can be taken as the length of the stick as measured in S. It would of course be exactly 1 metre.

Now consider the observer in S'. He too follows the rule that one must mark off both the ends at the same instant. For him, the events E_1 and E_2

are *NOT* simultaneous. But the pair of events E_3 and E_2 are simultaneous, as also the pair of events E_1 and E_4. Therefore, to measure the length of the stick in S, he determines either the distance E_3E_2 or the distance E_1E_4. This length will definitely be less than 1 metre. Figure 5.9 explains why, and this is where the calibration exercise carried out earlier is helpful.

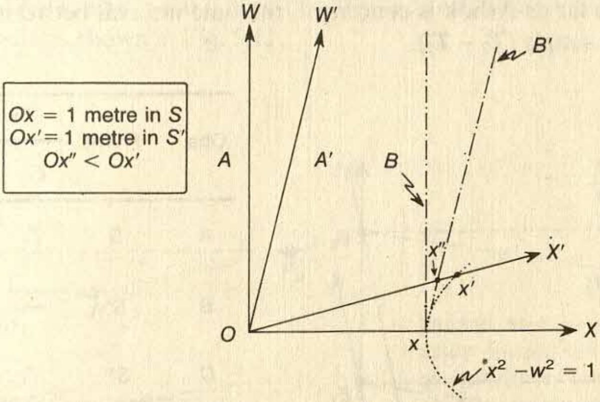
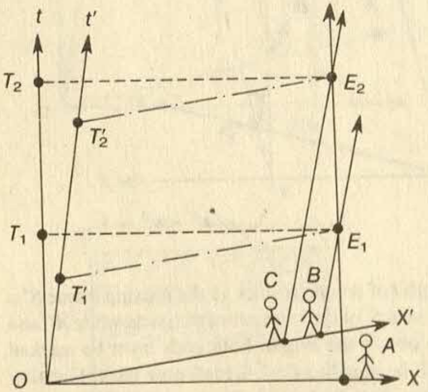


Fig. 5.9 This figure explains why the length l of the metre-stick in the moving frame S' is less than 1 metre. A and B are the world lines in S, of the two ends of the stick, while A' and B' are the corresponding lines in S'. To obtain the length, both ends must be marked simultaneously. Thus the length in S' will be equal to Ox''. Recall now the calibration exercise of Fig. 5.2. The dotted line is the hyperbola for 1 metre. Obviously, Ox'' is less than Ox', i.e., 1 metre. The stick therefore appears contracted when viewed in S'.

Now suppose that the chap in S is trying to measure the length of a metre-stick in S'. You would think that he would obtain a length which is greater than 1 metre. No! On the contrary, he too would get a length *less* than one metre, because for him, it is S' which is moving. In other words, each observer will claim that the other's metre scale is shorter! Once again there is a symmetry which it is important to note. The message as far as we are concerned is that moving rods appear to contract. This is nothing but Lorentz contraction and comes out quite naturally in the Special Theory.

5.13 Time dilatation

I now come to the second important consequence namely, *time dilatation*. I have already hinted about it while discussing moving clocks. Let me now amplify. As before, we have two inertial frames S and S', with S' moving uniformly at a speed v with respect to S. Ashok (A), Bala (B), and Chander (C) are three observers with Ashok in S, and the remaining two in S'. Each has a clock. E_1 and E_2 are two events (see Fig. 5.10), and the observers in the two frames are supposed to find out the time difference between these two events and then compare them. As far as A is concerned, in his frame the two events occur at the same place, i.e., the place where he is stationed. So, looking at his clock, he reads the time T_1 when the event E_1 occurs, and then the time T_2 when the event E_2 occurs. Thus, as far as Ashok is concerned, the time interval between the two events is simply $(T_2 - T_1)$.



Obs.	Frame	Time	
		E_1	E_2
A	S	T_1	T_2
B	S'	—	T'_2
C	S'	T'_1	—

$$(T_2 - T_1) \neq (T'_2 - T'_1)$$

Fig. 5.10 Illustration of time dilatation. E_1 and E_2 are two events observed both in S (fixed frame) and in S' (moving frame). While E_1 and E_2 occur at the same spot in S, they occur in different places in S'. Thus, in S they may be observed by A, but two persons (B and C) are required in S'. The time difference between E_1 and E_2 measured in the two frames are not the same. By further considerations, one can show that $(T'_2 - T'_1)$ is less than $(T_2 - T_1)$.

Let us now examine the situation in S'. Here things are somewhat different. Bala is nicely positioned to observe the event E_1, because his world line (in S' that is) passes through E_1. But he cannot observe the other event E_2. However, that is not a problem for Chander can take care of that. Thus, between B and C the two are able to measure the time T'_1 and T'_2 corresponding to the two events in the frame S'. They then have a little conference between themselves and then tell A that the time separation between the events is $(T'_2 - T'_2)$, and not $(T_2 - T_1)$ as A claims it is. So there is a dispute. But we know both are correct!

This leads us to the *relativity of simultaneity*. Once again there are two inertial frames S and S', with S' moving uniformly with a velocity v with respect to S. Q_1, Q_2 and R_1, R_2 are four events as in Fig. 5.11, and they are observed in both the frames. As is clear from the figure, the events Q_1, Q_2 are simultaneous in S but not so in S'. Likewise, the events R_1, R_2 are simultaneous in S' but not so in S. Moral: *What is simultaneous for one observer need not be so for another*! Remember that the two observers are in two different inertial frames which are in uniform relative motion. However, the differences are spectacular only when v is nearly equal to c. That is why in our day-to-day life we do not observe anything similar to what is shown in Fig. 5.11.

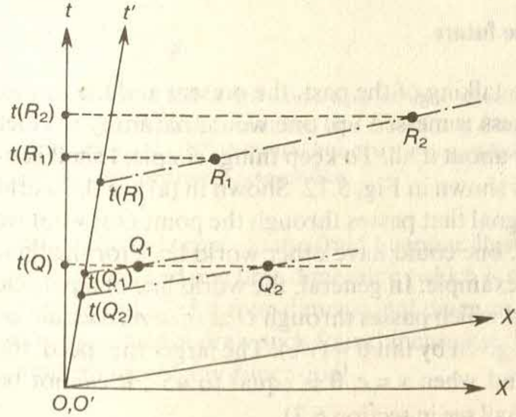


Fig. 5.11 Meaning of simultaneity in special relativity. Q_1 and Q_2 are two events which are simultaneous in S but not so in S'. R_1 and R_2 are two other events with just the opposite characteristics, i.e., they are simultaneous in S' but not in S.

Once again I repeat that both, length contraction and time dilatation follow automatically from the Lorentz transformations.

5.14 About proper things

There are many lessons to be drawn from what I have said so far. In all our discussion, we always had two inertial frames S and S', with one of them moving relative to the other. The frame that is fixed, at least in our imagination, is called the *proper frame*. We can also think of it as a frame in which there is an object which is at rest with respect to that frame (for example the metre-stick in S which I discussed earlier). The length of a rod which is at rest in the proper frame is called the *proper length*. Likewise, the *proper time* interval is the time interval measured at some fixed space point in a particular frame of reference. For example, the time interval measured by Ashok (see Fig. 5.10) would be a proper time interval. Time interval between two events measured using *two* different clocks (i.e., at two different space points) is not a proper time interval. Thus the time interval measured jointly by Bala and Chander is not a proper time interval.

To summarise:

1. The length of a rod is the highest when measured in a proper frame; the rod appears contracted in other frames.
2. The time interval between two ticks of a clock is the smallest when measured in the proper frame. If the events corresponding to these ticks are observed in a frame that is not proper, then the corresponding time interval is longer.

5.15 Past, present and the future

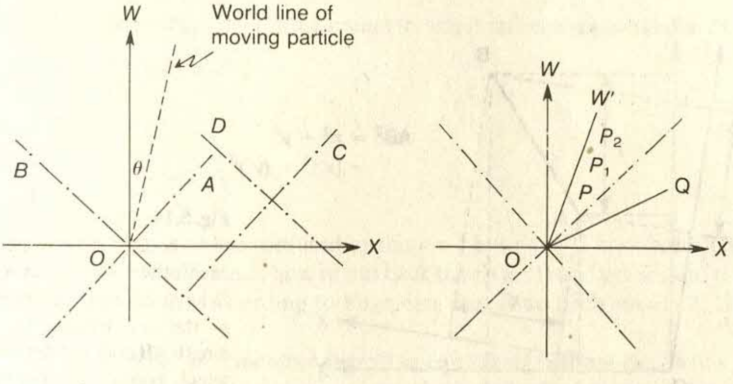
Normally, we are used to talking of the past, the present and the future. Now that the time business is messed up, one would naturally wonder what relativity has to say about it all. To keep things simple, I shall consider only one frame S as shown in Fig. 5.12. Shown in (a) are the world lines A and B of a light signal that passes through the point O at what we call zero time. Of course, one could have other world lines for the light signal, like C and D, for example. In general, the world line of a particle travelling at a speed v and which passes through O at time zero would be a line making an angle θ, given by $\tan \theta = (v/c)$. The larger the speed, the greater is the angle θ, and when $v = c$, θ is equal to 45°; it cannot be greater than 45° (as we shall see in section 6.3).

Consider now a point P in the shaded area of Fig. 5.12(b), and through it draw the line OP. This line OPW' could be the time axis of some suitable inertial frame S' say. For this frame S', events O and P would occur at the same space point but at different times. What is true of P is true of all points in the shaded region. So this region is called *time-like*.

Now consider the events P, P_1, P_2. All these lie on the same W'-axis. It is clear that P_1 is later to P and P_2 is later to P_1. This enables us to identify the future. Similarly, we can also identify the past.

Next we consider a point Q in the unshaded region—see (b). The line OQ could be the space axis of a suitably chosen inertial frame S'' say. Events O and Q now represent simultaneous events taking place at different space points. Therefore, the entire unshaded region is called *space-like*.

For simplicity, I have so far considered the one space dimension plus one time dimension case. The space-time diagram for the 2 + 1 case is shown in Fig. 5.13. Instead of the lines $x = ct$ and $x = -ct$, we now have cones separating the time-like and the space-like regions. This cone is



(a) (b)

Fig. 5.12 (a) A and B are world lines of light signals propagating in the $+X$ and $-X$ directions, and which meet at O at time $t = 0$. C and D are other examples of world lines of light signals. (b) OW' the world line of a moving particle. The shaded region is time-like and the unshaded region is space-like.

called the *light cone*. I am afraid I cannot illustrate the case of three space dimensions and one time dimension (which is the one most relevant to us). This would need a four-dimensional diagram, and we three-dimensional creatures cannot draw such four-dimensional diagrams. But we can do the mathematics of any dimension!

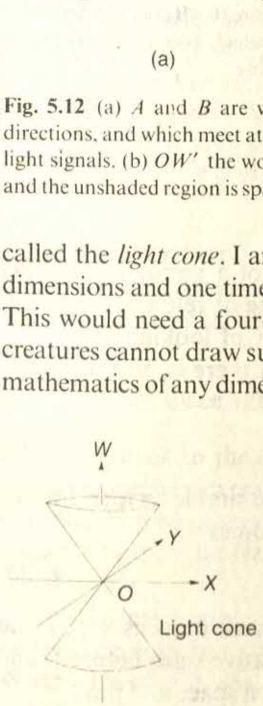


Fig. 5.13 The previous figure deals with the (1 space + 1 time) dimension case. Shown here is a similar partition for the (2 space + 1 time) dimension case. The partition into space-like and time-like regions is effected by the light cone.

5.16 What is left invariant by Lorentz transformations?

There is an interesting geometric interpretation that can be given to Lorentz transformations. To start with, we consider an ordinary rotation of axes as in Fig. 5.14. It is obvious that although the components of the vector **AB** in the two coordinate frames (OX, OY) and (OX', OY') are different, the length of **AB** as computed in both is the same. All I am saying is,

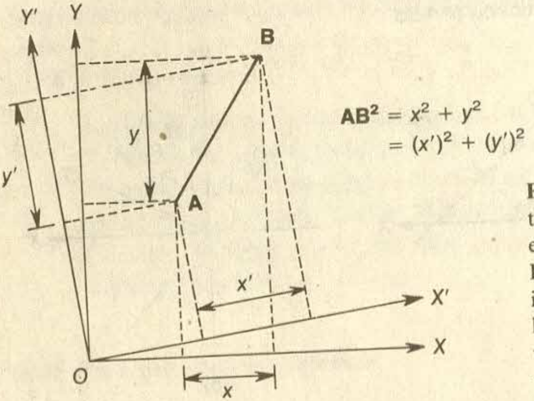


Fig. 5.14 Two coordinate systems rotated with respect to each other. The points **A** and **B** have different components in the two frames, but the length **AB** does not depend on which coordinate system we use.

$$x^2 + y^2 = \mathbf{AB}^2 = x'^2 + y'^2. \tag{5.10a}$$

Similarly, in three dimensions we have

$$x^2 + y^2 + z^2 = x'^2 + y'^2 + z'^2. \tag{5.10b}$$

In short, (5.10) merely says that the length of a vector in ordinary space is left invariant by a rotation of the frame of reference. Analogously, we now ask: "Is there a geometrical way of looking at Lorentz transformations, similar to (5.10)?" It turns out there is, and the quantity that is left invariant is $(x^2 + y^2 + z^2 - c^2t^2)$, i.e.,

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2. \tag{5.11}$$

We can make (5.11) look like (5.10) by using two simple recipes. Firstly, we put $c = 1$ (see Box 5.2) whereupon (5.11) becomes

$$x^2 + y^2 + z^2 - t^2 = x'^2 + y'^2 + z'^2 - t'^2. \tag{5.12}$$

(Remember t and t' are now measured in the same units as x, y, z and x', y', z'.) We still have a problem with the negative signs before t^2 and t'^2 but they really highlight the difference between space and time.

Now for our second recipe which is to replace t with $-iW$ and t' with $-iW'$. Here $i = -1$. With this definition, $W = it$ and $W^2 = -t^2$ since $i^2 = -1$. Using this, (5.12) now becomes

$$x^2 + y^2 + z^2 + W^2 = x'^2 + y'^2 + z'^2 + W'^2. \tag{5.13}$$

This result now looks like (5.10) and is nothing but a statement of Pythogoras' theorem in four-dimensional space-time. To put it differently, *Lorentz transformations preserve space-time intervals or intervals between events in space-time*. Modern day theories of physics pay much

Box 5.2 When we put $c = 1$, the Lorentz transformation equations read:

$$\begin{aligned} x' &= (x - vt)/\sqrt{1 - v^2} \\ y' &= y \\ z' &= z \\ t' &= (t - vx)/\sqrt{1 - v^2} \end{aligned} \tag{1}$$

It is quite likely you are confused by this $c = 1$ business—I have been! The worry that one usually has is how to put back the c's in (1) and get back to the standard form (5.6)? According to Feynman, that is no problem at all. He says:

> It is much easier to remember them [the equations] without the c's in them, and it is always easy to put the c's back, by looking after the dimensions. For instance, in $(1 - v^2)$, we know that we cannot subtract a velocity squared, which has units, from the pure number 1, so we know that we must divide v^2 by c^2 in order to make that unitless, and that is the way it goes.

attention to what is referred to as *Lorentz invariance*—basically it simply means that the physics underlying the theory must remain the same under Lorentz transformations.

5.17 Absolute space again

Let us get back to the absolute space proposed by Newton. When the subject of mechanics really got going, people realised that relative motion is all that can be observed and that there was really no use for absolute space. Why then did Newton go through the trouble of inventing such a space?

Newton was a rather religious person. He actually started out as a student of theology but got "side tracked" into physics and mathematics. What a blessing it was! But after his famous *Principia*, theology again began to dominate his interests. It has been suggested that Newton postulated absolute space and absolute time so as to reflect God's universality, His eternity and His infinity.

The idea of absolute space came under strong attack even during Newton's lifetime but Newton clung to his views. To him, denial of absolute space meant the denial of God. To demonstrate the existence of such a space, he proposed the rotating-bucket experiment illustrated in Fig. 5.15. Due to the rotation of the bucket, the water surface will take on a concave shape which can be clearly distinguished from the plane

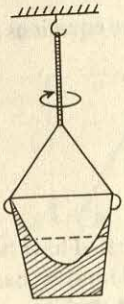


Fig. 5.15 Rotating-bucket experiment of Newton.

surface of water that is at rest (in a non-rotating bucket). Does this not prove that rotational acceleration can always be detected and is therefore absolute? Newton thought it did.

Nearly two centuries later, Ernest Mach questioned this view. His argument went somewhat like this: Supposing the Universe had just two spherical bodies, say two planets as in Fig. 5.16, one rotating and the other stationary. If each planet is observed from the other, the observers cannot decide amongst themselves as to which planet is rotating and which is not. But wait a minute! Will not the rotating body bulge at the equator and flatten at the poles? This is a fact of life, and surely such bulging and flattening can be used to detect absolute rotation, absolute acceleration and therefore by implication, absolute space also. It is precisely here that Mach raised a fundamental question. Whoever has done an experiment in a Universe that is empty but for two planets? Since such an experiment has not been done, how can we conclude that the rotating body would be squashed? Well, if Mach is so clever, then, how does he explain the fact that, in the real Universe we live in, many planets and celestial bodies are bulging at the equator and flattened at the poles. That, said Mach, was due to the *distant masses*.

In the real Universe, whatever be the stellar body we select, it is *always* surrounded by innumerable other celestial bodies located at enormous

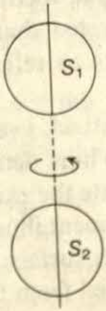


Fig. 5.16 Illustration of a hypothetical Universe containing just two spherical masses, one stationary and the other rotating.

distances from the chosen object. These distant masses move ever so slowly with respect to the object under consideration that they effectively provide a fixed frame of reference. Thus, wherever we are in our Universe, the distant stars always provide a convenient reference frame. It is the gravitational force due to that distant matter that causes water in a rotating bucket to assume a concave shape, planets to bulge, etc. In other words, according to Mach, there is no such thing as absolute acceleration in the sense of Newton. By corollary, absolute space also becomes an undefinable and undefendable concept. Today, we believe in Mach rather than in Newton.

To sum up:

- Einstein's Special Theory of Relativity is based on two postulates: (i) there is no preferred inertial frame, and (ii) the speed of light is a constant.
- Galilean transformations do not satisfy the second postulate of Einstein while Lorentz transformations do.
- Lorentz transformations are thus consistent with the finding of Michelson and Morley.
- Lorentz transformations mix space and time coordinates in such a manner that the time coordinate in one inertial frame is related to the space as well as the time coordinate of another inertial frame.
- An event is an occurrence at a particular point in space at a particular time. World or space-time is simply a collection of all possible events. The locus of events relating to a given particle is the world line of that particle.
- A rod which is moving, appears contracted.
- Moving clocks run slow. It is impossible to build a clock that does not run slow when moving. If such a clock can be built, then an observer in an inertial frame can determine his own velocity which, however, is not permitted.
- Events that appear simultaneous in one inertial frame need not appear so in another.
- In special relativity, in each inertial frame one can talk separately of space and of time, but the space of one inertial frame cannot in general be related purely to the space of another. Same with time.
- Related to the above is the fact that in Einstein's relativity, only intervals of space-time, i.e., distances between events are preserved during Lorentz transformations.
- Newton had absolute space and absolute time. Einstein changed that; there is now only space-time.

6 *Relativistic Kinematics*

So far we have focussed on how to relate events in two inertial frames which are moving uniformly with respect to each other. We now move on to a few practical problems.

6.1 About velocities

Velocity, momentum and acceleration are central concepts in mechanics. In Newtonian mechanics, the different inertial frames are related to each other via Galilean transformations. Now that we have decided to give up the Galilean transformations in favour of the Lorentz transformations, we must ask: Whatever happens to Newton's laws under the new circumstances? I won't consider Newton's laws as such, but some related concepts like velocity, momentum and energy.

We start with velocity. Let S and S' be two inertial frames, moving relative to each other with a velocity v. To fix our ideas let us say that S corresponds to a railway track and S' to a train. An observer on the ground near the railway track (he obviously belongs to the frame S) sees a passenger move inside the train. With respect to the train, the passenger is moving with a velocity u' say—see Fig. 6.1—but to the person on the ground, the passenger would appear to move with a velocity

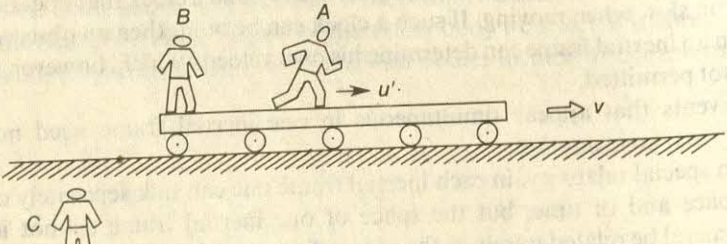


Fig. 6.1 Illustration of the velocity-addition theorem. A railway carriage is moving at a speed v, and A walks on it with a speed u'. To B who is also riding the same carriage, the speed of A appears to be u' but to C who is on the ground the speed of A is different from u According to Newton, C would measure a speed u given by formula (6.1) whereas according to Einstein, the speed of A, as it appears to C would be given by formula (6.2). When v is small, Einstein's formula gives practically the same answer as that of Newton.

$$u = u' + v, \tag{6.1}$$

that is if we believe in Newtonian relativity. This result follows from common sense. But according to Einstein,

$$u = (u' + v)/\{1 + (u'v/c^2)\}, \tag{6.2}$$

a result which is called the *relativistic velocity-addition theorem*. Compare (6.1) and (6.2); the two differ on account of the denominator in the second equation. When v is small, we can neglect the term $(u'v/c^2)$ in the denominator of (6.2) whereupon we get $u = (u' + v)$, which is the same as (6.1). However, when v is nearly equal to c, the denominator in (6.2) makes a world of difference.

Suppose that instead of a passenger moving in a train, we have a light pulse propagating inside. According to (6.1), the man on the ground would see this pulse moving with a velocity $(c + v)$, which is greater than c. But this cannot be. Let us now see what (6.2) gives. This predicts that the chap on the ground would also see the pulse travelling with a speed c. In other words, (6.2) is consistent with the second postulate of Einstein's relativity.

In our usual studies in mechanics, v is quite small compared to c. As we just saw, formula (6.2) then predicts $u \sim (u' + v)$. So, (6.2) not only respects the second postulate of Einstein, but also gives the simple result of Galileo/Newton if we are lazy and would like to neglect v compared to c. In other words, this formula shows why the Newtonian formula works when v is small compared to c.

6.2 Relativistic mass

We next turn to momentum. Newton said momentum = constant × velocity, the constant being identified as the (inertial) mass. Einstein agrees that momentum equals mass times velocity but with *the mass depending on the velocity*, according to the formula

$$m = m_0/\{1 - (v/c)^2\}^{1/2}. \tag{6.3}$$

When the velocity v of the object is zero, $m = m_0$. The latter is therefore called the *rest mass* of the object. Result (6.3) is very important. According to Newtonian mechanics, the mass of an object is the same for two observers in two inertial frames moving with a uniform velocity with respect to reach each other; but now, the two observers see different masses!

Figure 6.2 shows how m changes with the velocity. From it, two things are clear. Firstly, if the velocity is small, the mass hardly changes. Thus, when a cricket ball is hit hard and speeds towards the boundary,

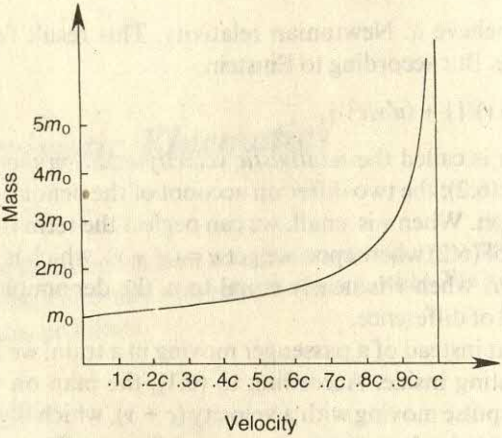


Fig. 6.2 Variation of mass with velocity, as predicted by Einstein.

its mass is certainly increased but only by an insignificant amount. Secondly, when $v = c$, the mass becomes infinity! Now you may ask: "Light quanta or photons travel at the speed c. Does that mean that photons have infinite mass?" No, because the rest mass of the photon is zero. A better way of saying this is that the photon is never at rest; it always moves, and does so at the speed c. Note, however, that *although the photon has no mass, it certainly has both momentum and energy*.

The velocity of light plays the role of a *ceiling* velocity in physics. The explanation of the latter is a bit involved. Basically it goes like this. Let E_1 and E_2 be two events with E_2 following E_1. E_1 may be the lighting of a wick, and E_2 the explosion of a cracker. Observers in different inertial frames might disagree about the time interval between the two events, but no one would say that the cracker exploded first and then the wick was lit. Clearly, event E_1 is the cause, and event E_2 is the effect. *Effect always follows the cause.* This is the *law of causality* and is not altered when going from one inertial frame to another. Physicists have immense faith in this law. If velocities greater than c are permitted, then we would have trouble with causality.

To understand this last remark, consider a frame A in which there is an object moving with a velocity v_A. B is another frame, moving with a velocity v with respect to frame A and in such a manner that v is directed opposite to v_A—see Fig. 6.3. For an observer in frame B, the velocity of the object in A would appear as

$$v_B = (v - v_A)/\{1 - (vv_A/c^2)\} \qquad (6.4)$$

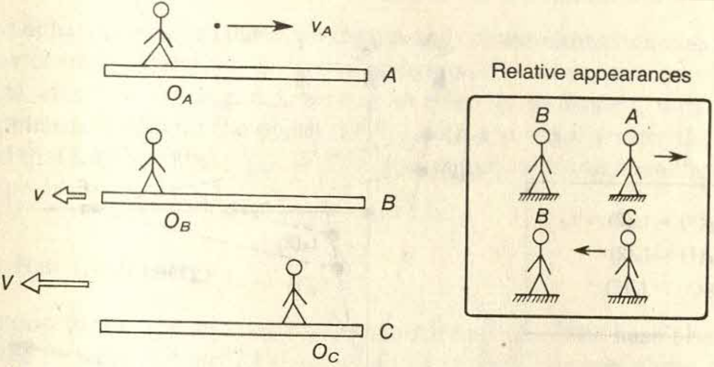


Fig. 6.3 Shown here are three frames A, B and C, used for arguing why velocities greater than c are not possible. v_A is the velocity of an object moving in A.

(see formula (6.2)). To complete the story, let us also consider another frame C which is moving relative to A and B as shown in Fig. 6.3.

We now come to the causality violation business. We assume for the sake of argument that v_A is *greater* than c. We further suppose that v is such that (vv_A/c^2) equals unity. Then the denominator in (6.4) vanishes, making v_B equal to infinity. What it means is that under these conditions, the world line of the moving object in the B frame would be a horizontal line as in Fig. 6.4. Let us say this moving object carries a light which is flashed a couple of times. Call these flashing events E_1 and E_2. It is clear from Fig. 6.4 that to the observer in frame B, these events would be simultaneous. What about frame A? Here, event 1 occurs *before* event 2 which is OK. On the other hand, in frame C, event 2 occurs before event 1, i.e., the *sequence is reversed*. If E_1 is the cause and E_2 the effect, then in frame C the *cause occurs after the effect* which is absurd. How did this absurdity arise? It arose because we assumed v_A could be greater than c. Since causality is sacred, we regretfully must conclude that velocities greater than c are not possible.

In passing I should also mention that Professor E.C.G. Sudarshan has proposed the existence of particles named *tachyons* which can travel faster than light. As yet there is no experimental evidence for them. You might wonder about the violation of Einstein's theory. That is a problem isn't it? It turns out that there is an interesting solution to the problem. First, since tachyons travel with a speed greater than c, their frame gets represented as in Fig. 6.5(b) instead of as in Fig. 6.5(a), the scenario we are usually accustomed to. The net result is that whereas our world lines are confined to the time-like region (see Fig. 6.5(c)), those of tachyons are confined to the space-like region. The light cone acts like a

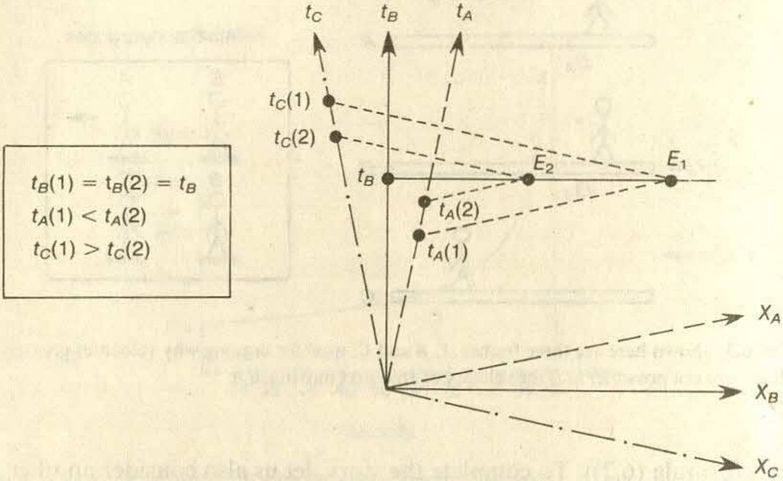


Fig. 6.4 Space-time diagrams of the three frames of the previous figure. The horizontal line is the world line of the object (moving) in *A*, as it appears to an observer in *B*. E_1 and E_2 are two events on this world line, the former being the cause and the latter the effect. To the observer in *A*, causality is respected but to the observer in *C* the effect occurs before the cause which of course is not possible.

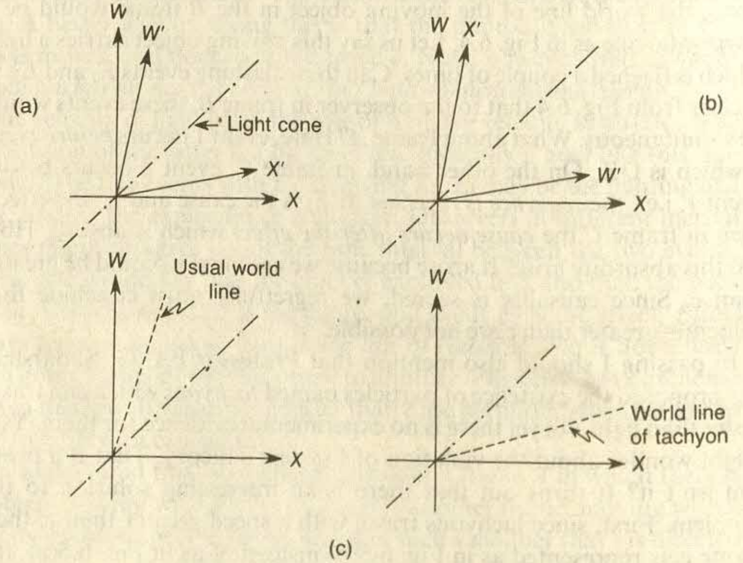


Fig. 6.5 (a) and (b) show the space-time diagrams for a frame moving with velocities $< c$ and $> c$ respectively. For tachyons, the situation would be as in (b). With respect to the light cone, our world lines and those of tachyons would be as in (c).

sort of barrier between these two regions and communications across this barrier are *not* permitted. So even if tachyons exist, we cannot communicate with them! In Fig. 6.3, we had an observer in frame C trying to "communicate" with the object moving with a velocity greater than c; and that led to an absurdity. If there is no communication, then there is no problem.

6.3 Relativistic energy

We now turn to the relativistic expression for energy. Who hasn't heard of the formula $E = mc^2$? Let us get a feel for how it comes about and what it means.

According to Newton, the kinetic energy K of a particle of mass m_0 and velocity v is given by

$$K = (1/2) \cdot m_0 \cdot v^2. \tag{6.5}$$

This formula is very familiar to us. Relativity on the other hand says

$$K = mc^2 - m_0c^2. \tag{6.6}$$

If you care for a bit of calculus, then peep into Box 6.1 to see how this result is obtained.

Box 6.1 Consider a particle of mass m_0 acted on by a force F. Let the particle be initially at rest and let F accelerate it to a velocity v. According to Newton, the kinetic energy would be given by

$$\begin{aligned} K = \int_{\text{vel}=0}^{\text{vel}=v} F dx &= \int_0^v m_0 \frac{dv}{dt} dx \\ &= \int_0^v m_0 \, dv \frac{dx}{dt} \\ &= m_0 \int_0^v v dv \\ &= \frac{1}{2} m_0 v^2 \end{aligned} \tag{1}$$

In special relativity we are a bit more careful and write

$$K = \int_0^v F dx = \int_0^v \frac{d}{dt}(mv) dx$$

$$= \int_0^v d(mv)\,\frac{dx}{dt}$$

$$= \int_0^v (mdv + vdm)v$$

$$= \int_0^v (mvdv + v^2dm). \qquad (2)$$

Now

$$m = m_0/\sqrt{(1 - v^2/c^2)}.$$

Therefore,

$$m^2c^2 - m^2v^2 = m_0c^2$$

or

$$(mvdv + v^2dm) = c^2dm. \qquad (3)$$

Substituting (3) in (2), we get

$$K = \int_{m_0}^{m} c^2dm = c^2 \int_{m_0}^{m} dm = mc^2 - m_0c^2$$

which is equation (6.6).

Rearranging (6.6) we get

$$mc^2 = m_0\,c^2 + K. \qquad (6.7)$$

Writing E for $(m_0c^2 + K)$, we obtain

$$E = mc^2, \qquad (6.8)$$

the famous result. What (6.8) says in effect is that the *total* energy of the particle is the sum of its kinetic energy and its *rest energy* m_0c^2.

Equation (6.8) is innocent looking but has deep implications. Normally we think that a particle has no energy if it is not moving. True it could have some potential energy on account of sitting on top of a mountain, for example. But suppose our particle is deep in outer space where it experiences practically no gravitational force; also that the particle has no other energy whatsoever, like magnetic or electrostatic energy. And further, it is not moving. Now what? Newton would say: "The particle has no energy." Einstein would say, "Wrong! The particle has a rest energy of m_0c^2." The larger the rest mass m_0, the greater is this *rest energy*. Thus in relativity, *mass and energy are related.*

6.4 Mass–energy equivalence

We have heard of many kinds of energy before—kinetic energy, potential energy, electrostatic energy, strain energy, and so on. We also know that energy in one form can be converted into another. What about this new relation (6.8)? Can pure mass be converted into energy? Conversely, can energy be converted into mass? Both are possible and have been demonstrated.

Nuclear reactions provide a clear and convincing evidence for the conversion of mass into energy. Consider the nuclear reaction

$$D + D \rightarrow P + T + \text{energy} \tag{6.9}$$

—see also Fig. 6.6. The D-D reaction is just like the chemical reactions we are used to; the only difference here is that the participants are atomic nuclei and not atoms or molecules as in chemical reactions. Quite a bit of energy is released in the reaction (6.9). Where does this energy come from?

Now nuclear physicists have very painstakingly measured the rest

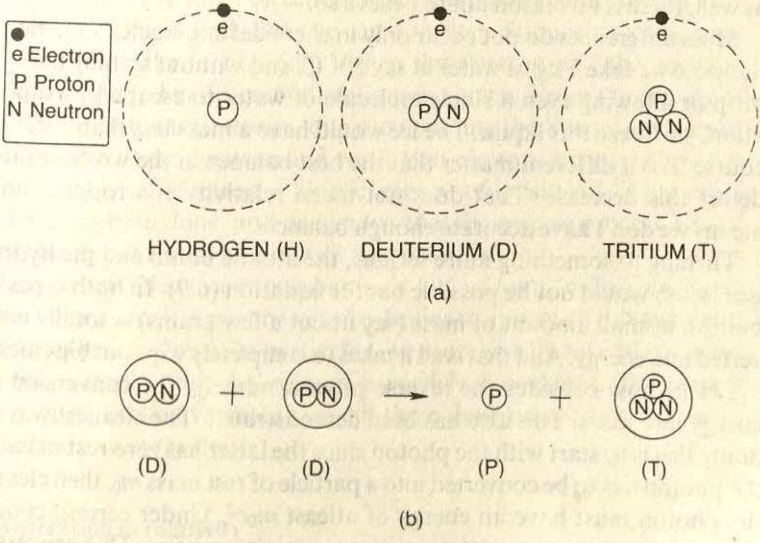


Fig. 6.6 (a) shows the isotopes of hydrogen. In all cases, there is only one proton. The difference between isotopes lies in the number of neutrons in the nucleus. Isotopes of an element have very similar chemical properties but very different nuclear properties. (b) illustrates the reaction (6.9).

masses of many many nuclei, including P, D, and T. From the tables of nuclear masses we have:

rest mass of P = 1.0078252 a.m.u.

rest mass of D = 2.0141019 a.m.u.

rest mass of T = 3.0160494 a.m.u.

Normally, we express mass in terms of grams or kilograms. But a gram is too large a unit for expressing the mass of nuclei. So one uses a different unit namely the a.m.u. which stands for *atomic mass unit* (one a.m.u. = 1.66×10^{-24} gm.). In terms of this unit, the mass of 2D is 4.0282038 while the mass of T plus P is 4.0238746. The difference is 0.0043292 a.m.u. This much of mass is *destroyed* and appears as energy during the D-D reaction. Careful experiments have confirmed that precisely this much energy is liberated in the reaction (6.9). The conversion of mass into energy has been demonstrated any number of times in nuclear reactions.

You might now ask: "Should there not be such a mass difference after chemical reactions?" Indeed there is, but the difference is about a million times smaller than what we have in nuclear reactions. At present our accuracy in measuring masses is not good enough to detect such small differences. Meanwhile, we believe that (6.9) operates for chemical reactions as well; there is no reason not to believe so.

Mass differences do not occur only in chemical and nuclear reactions. Suppose we take 1 kg of water at say 50° C, and without spilling a single drop or allowing even a single molecule of water to escape by evaporation, we freeze this liquid. The ice would have a mass less than 1 kg! Of course, it is a different matter that the best balance in the world cannot detect this decrease. That does not mean relativity is wrong; it only means we don't have accurate enough balances.

Turning to something more serious, the atomic bomb and the hydrogen bomb would not be possible but for equation (6.9). In both types of bombs, a small amount of mass (say about a few grams) is totally converted into energy. And that is all it takes to completely wipe out big cities!

Let us now consider the reverse process namely, the conversion of energy into mass. This also has been demonstrated. The cleanest way of doing this is to start with the photon since the latter has zero rest mass. If the photon has to be converted into a particle of rest mass m_0, then clearly the photon must have an energy of atleast m_0c^2. Under certain conditions, photons can materialise as electron-positron pairs. This process is known as *pair production*.

As you probably know, the positron has the same mass as the electron but the opposite charge. The simplest conversion reaction possible is high energy photon → electron + positron.

This reaction can be arranged so as to balance energy. It also balances charge since the total electric charge both before and after the reaction is zero. But there are problems in balancing the momentum. In short, the above reaction is ruled out. However, with cooperation from another "spectator" (which could be a nucleus, for example), *all* the rules of the game could be duly followed and pair production achieved.

Pair production can be demonstrated using a *cloud chamber*, which is simply a big vessel containing saturated water vapour that is ready to condense—see Fig. 6.7(a). To observe the condensation, a window (covered with a glass plate) is provided. If we want to create an electron-positron pair, the photon energy must be greater than $2m_ec^2$, where m_e is the rest mass of the electron. Let us say we have a beam of such photons entering the cloud chamber. These are high energy photons, and can get into the chamber by penetrating the walls. If one of these photons creates a pair, the particles so produced would leave a trail while moving in the chamber, just like jet planes do while flying high. Incidentally, only charged particles leave condensation trails in cloud chambers; the photon, for example, cannot. So we should expect something like in Fig. 6.7(b). Here the trails of the electron and the positron overlap because both take off in the same direction, and we are not able to say for sure that both were produced. But one can do better. Supposing a magnetic field is applied like in Fig. 6.7(c); then the electron and the positron would be deflected in different directions as shown. From the density of the droplets in the trajectories, one can compute the kinetic energies of the electron and the positron and verify that photon energy = $\{2m_ec^2$ + kinetic energy of electron + kinetic energy of positron$\}$. This cross checking has been done, and one knows that the reverse process, i.e., of energy being converted into mass is also possible.

In summary, Newtonian mechanics is OK as long as the velocity of the particles is small. Once the velocity begins to approach c, Newtonian mechanics breaks down. In particular, we must give up several old ideas like, for example, that the mass of a particle is constant. The special theory gives explicit formulae for all these deviations from the traditional Newtonian results.

6.5 Aberration in relativity

Recall earlier attempts to understand aberration in terms of the ether. Since the ether idea has now been dropped, it may be asked: "What happens to stellar aberration now?"

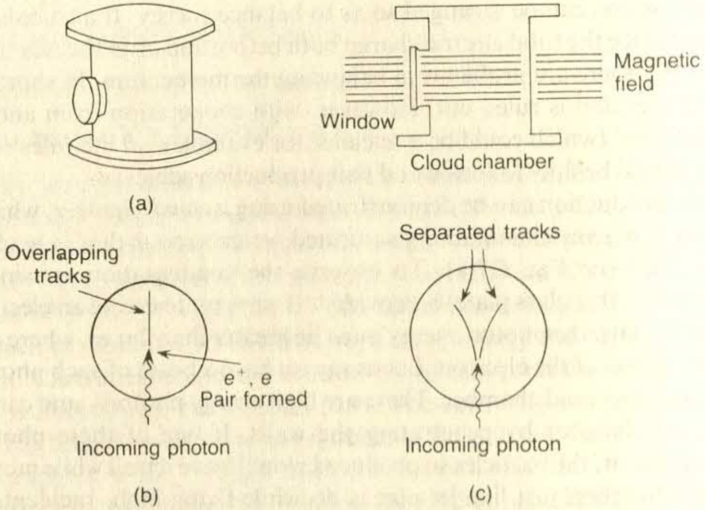


Fig. 6.7 This figure illustrates the demonstration of the conversion of energy into mass. The observations are made using a cloud chamber (a). Being uncharged, the photon does not leave any track but the electron and the positron do. The tracks are viewed through a glass window. (b) shows the appearance of tracks when no magnetic field is applied. The electron and the positron both fly off in the same direction, and their tracks overlap; they cannot therefore be seen apart. To overcome this difficulty, a magnetic field is applied as in (c) whereupon, the electron and the positron are deflected in different directions.

Consider two inertial frames *S* and *S′* as in Fig. 6.8, with a source of light at *O′*. Consider a light ray emerging from *O′* and making an angle θ' with the direction of motion. Supposing there is an observer *A* at *O* in *S* and that this ray is emitted when *O′* just passes *O*. For him, the ray would appear to make an angle θ which is different from θ'. Using Lorentz transformations and doing a bit of algebra, one can show

$$\tan\theta = \frac{\sin\theta \times \sqrt{1-\beta^2}}{\cos\theta - \beta}\,. \tag{6.10}$$

Equation (6.10) relating the angles θ and θ' is called the relativistic equation for the aberration of light.

Stars move with velocities which are rather small compared to *c*; so β will be rather small and β^2 can be neglected. Now let us suppose that *A* is looking at a star which is supposed to be directly overhead. Then $\theta = 270^\circ$ (see Fig. 6.8(b)) which means $\cos\theta = 0$ and $\sin\theta = -1$. From (6.10) we thus obtain,

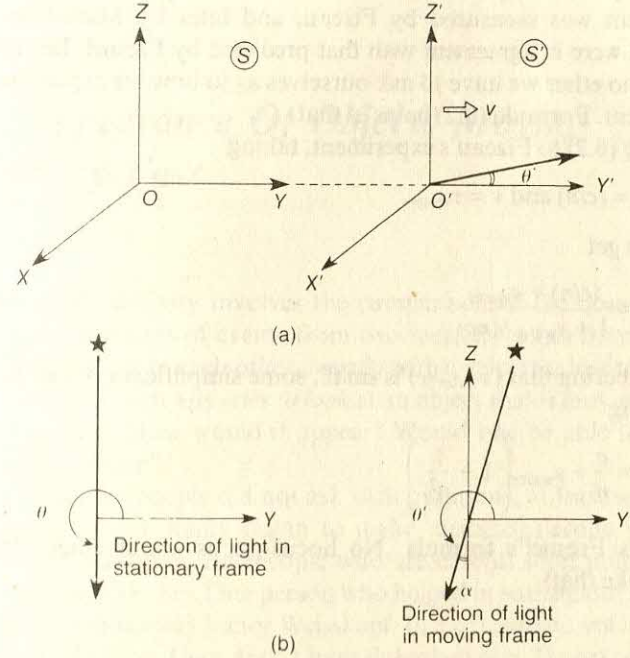

Z
S
X
O
Y
Z′
S′
v
θ′
Y′
O′
X′
(a)
θ
Y
Direction of light in stationary frame
Z
θ′
Y′
α
Direction of light in moving frame
(b)

Fig. 6.8 *S* and *S′* are two inertial frames of which *S′* moves uniformly with respect to *S* as in (a). θ′ is the angle which a ray of light emitted from *O′* makes with the *Y′*-axis. The same ray makes an angle θ with the *Y*-axis. The relationship between θ and θ′ is given by equation (6.10). (b) illustrates stellar aberration. The aberration angle α is computed as in (6.11).

$$\tan \theta' \sim (-1)/(-\beta) = c/v.$$

The aberration angle α one measures in the experiment is shown in Fig. 6.8(b). It is clear

$$\tan \alpha \sim \alpha = v/c \qquad (6.11)$$

which is the same result people had before special relativity. Notice, however, we do not bring in the ether. In short, special relativity enables us to explain all the earlier experimental results in a consistent manner, *without* bringing in the ether. No longer is there any conflict between the different results.

6.6 Fizeau's experiment in relativity

We next consider the Fizeau experiment discussed in Chapter 4. You will recall that Fresnel predicted a drag and also gave a formula for the drag coefficient. Fresnel based his theory on the ether, and the drag coefficient was measured by Fizeau, and later by Michelson. Their findings were in agreement with that predicted by Fresnel. But now that there is no ether we have to ask ourselves as to how we explain the drag coefficient. Formula (6.2) helps in that.

Apply (6.2) to Fizeau's experiment, taking

$$u' = (c/n) \text{ and } v = v_{\text{water}}. \tag{6.12}$$

We then get

$$u = \frac{(c/n) + v_{\text{water}}}{1 + v_{\text{water}}/(nc)} \tag{6.13}$$

Remembering that (v_{water}/c) is small, some simplifications can be made, obtaining

$$u = \frac{c}{n} + v_{\text{water}}\left(1 - \frac{1}{n^2}\right) \tag{6.14}$$

which is Fresnel's formula. No hocus-pocus about ether, drag and things like that!

7 *Appearance Of Objects Moving Very Fast*

A good bit of relativity involves the comparison of the observations made on the same set of events from two inertial frames moving uniformly with respect to each other, usually with a velocity close to that of light. What if one actually tries *to look* at an object that is moving with a speed close to c? How would it appear? Would one be able to see its Lorentz contraction?

For a long time people did not ask such questions, at least seriously, and when they did, many began to make mistakes despite years of experience in relativity! But people who are careful soon pointed out where such mistakes lay. One person who helped in sorting out the confusion that prevailed was Victor Weisskopf. In a companion volume *Why Are Things The Way They Are?*, I have described how Weisskopf made complicated physics seem simple for school students attending a course at CERN (see also Box 7.1). Today, one understands the appearance of fast moving objects much better, and I have seen a beautiful video film made on this subject.

Box 7.1 In an article entitled *The Visual Appearance of Rapidly Moving Objects*, Weisskopf says:

> We all believed that an object in motion appears to be contracted in the direction of motion by a factor $(1 - v^2/c^2)$. A passenger in a fast space ship, looking out of the window, so it seemed to us, would see spherical objects contracted to ellipsoids. This is definitely not so ... The reason is quite simple. When we see or photograph an object, we record the light quanta emitted by the object when they arrive simultaneously at the retina or at the photographic film. This implies that the light quanta have ***not*** been emitted simultaneously by all points of the object. The points further away from the observer have emitted their part of the picture earlier than the closer points. Hence, if the object is in motion, the eye or the photograph gets a "distorted" picture of the object, since the object has been at different locations when parts of it have emitted the light seen in the picture.

In special relativity, this distortion has the remarkable effect of cancelling the Lorentz contraction so that objects appear undistorted but only rotated. This is exactly true only for objects which subtend a small angle.

Let me give you a feel for how objects appear when they move at high speeds. Figure 7.1 shows the appearance of a rectangular grid when it is made to move with different velocities. In all cases, the observer is five units in front of the origin, i.e., rather close to the moving object. This is another way of saying that the object subtends a large angle at the eye of the observer. The more commonly discussed case is when the angle subtended is rather small. This is the case Weisskopf discusses, and we now consider one example in some detail.

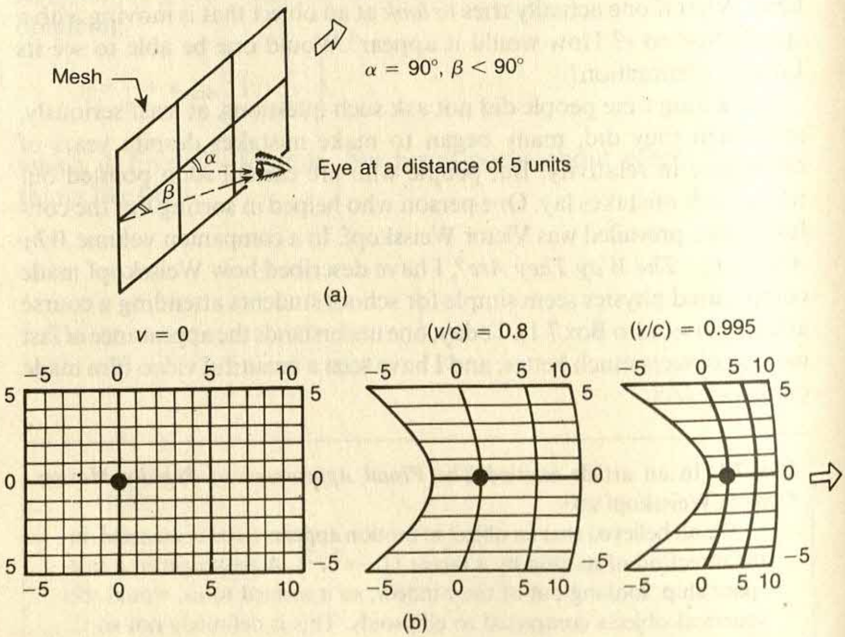


Fig. 7.1 The observer views a rectangular mesh in a direction perpendicular to that of motion, as shown in (a). He is at a distance of five units from the mesh. Notice that not all the rays coming from the mesh are perpendicular. The sketches in (b) show the appearance at different velocities.

Let us say there is a cube whose edge has a length L_0 in its own rest frame. This cube is moving at a speed close to that of light. The question is: How would it appear to an observer far away—see Fig. 7.2(a). We want the observer far away so that the light rays leaving the cube and reaching the observer can be regarded as parallel (in which case the angle subtended at the eye is zero). We shall now study the problem by considering three cases namely, (i) when the block is stationary, (ii) when the block is moving at a speed of roughly $c/2$ but that there are no effects due to special relativity, and (iii) when the speed is again $c/2$ but we allow for the effects of special relativity.

Case (i) is very easy; the observer would see only the face ABCD, and it would appear like a square as in Fig. 7.2(b). In case (ii) the observer would be able to see the two sides marked I and II. The reason for this is easy to understand.

Consider the light rays leaving the edge EH and travelling towards the observer. They have to cover an extra distance L_0 which would require a time L_0/c. But during this time, the cube would have moved a distance $\{v \cdot (L_0/c)\}$ along the x-direction. Light rays leaving AD (in this displaced position) now join those which left EH earlier, and together they all arrive simultaneously at the eye of the observer. In short, the observer would see what is shown in Fig. 7.2(c). He would see side I as a square but side II would look like a rectangle. You would think that case (iii) would be like case (ii); it partly is but there is a difference. This is because edges AB and DC are now Lorentz contracted, their lengths being reduced to $L = L_0 \cdot \sqrt{(1 - v^2/c^2)}$. So case (iii) is rather like case (ii), but with a bit of Lorentz contraction also thrown in—see Fig. 7.2(d). There is, however, an interesting point about the Lorentz contraction business.

Consider Fig. 7.2(e). This shows what a person would see if the cube were static but rotated by an angle θ about the line AD. Supposing the rotation is so adjusted that $\sin\theta = v/c$. Then the edges AE and AB would have lengths of $(L_0 v/c)$ and $L_0 \cdot \sqrt{(1 - v^2/c^2)}$ respectively. In other words, when a person sees the cube moving at a velocity v, special relativity would make the cube look *as if it is rotated* by an angle θ where θ is given by $\sin\theta = (v/c)$. Don't jump to the conclusion that there is no Lorentz contraction; it is simply not apparent. Rather, it makes the cube look *as if it is rotated.* Moral: Don't believe all that you see!

Another interesting case of visual effects refers to the so-called *headlight effect.* This is illustrated and explained in Fig. 7.3. This effect shows up dramatically in what is called the *synchrotron radiation.* Suppose we make electrons go round in a circular orbit at a speed close to c. This is routinely done in many accelerators all over the world. An electron in a circular orbit experiences an acceleration, and as you know an accelerated charged particle gives off electromagnetic radiation. Since the speed of the electron is nearly equal to c, there is a strong headlight effect. Thus tangential beams are emitted as the electron sweeps around—see Fig. 7.4.

The emitted radiation comprises of a whole spectrum of frequencies, but by careful design one can make any desired frequency have the maximum intensity. People make use of synchrotron radiation for various experiments in physics, chemistry and biology. There are many

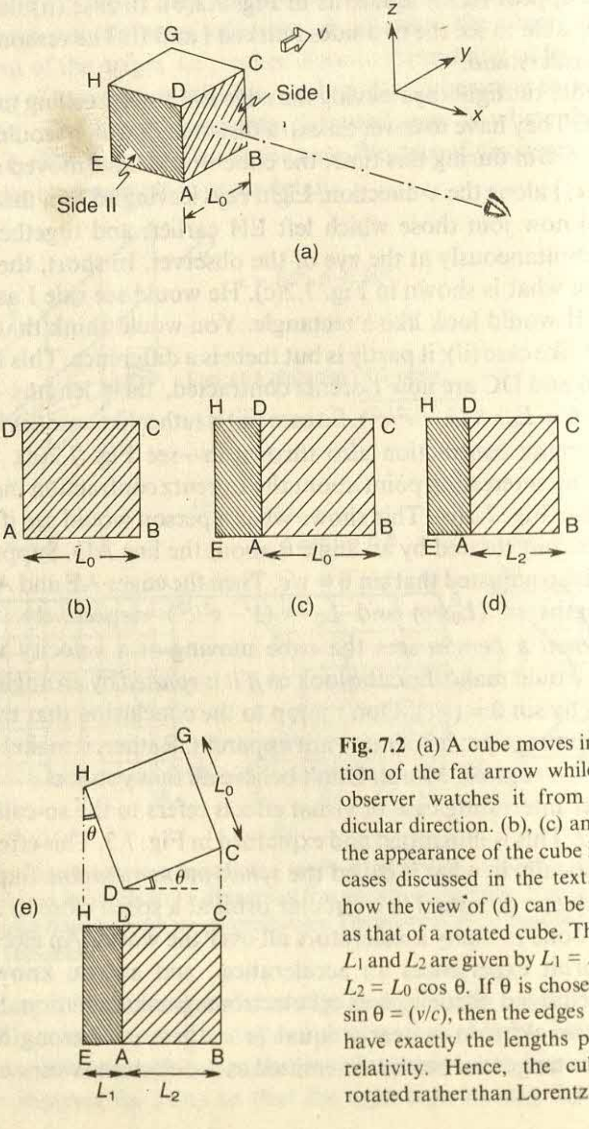


Fig. 7.2 (a) A cube moves in the direction of the fat arrow while a distant observer watches it from a perpendicular direction. (b), (c) and (d) show the appearance of the cube in the three cases discussed in the text. (e) shows how the view of (d) can be interpreted as that of a rotated cube. The distances L_1 and L_2 are given by $L_1 = L_0 \sin\theta$ and $L_2 = L_0 \cos\theta$. If θ is chosen such that $\sin\theta = (v/c)$, then the edges AE and AB have exactly the lengths predicted by relativity. Hence, the cube appears rotated rather than Lorentz contracted.

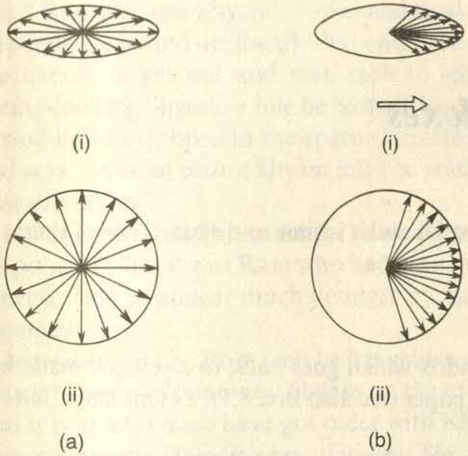


Fig. 7.3 Schematic figure illustrating the headlight effect. a(i) shows rays of light emerging from a static lamp. To emphasise that the rays diverge equally in all directions, the top view of the scene is shown in a(ii). b(i) and b(ii) are the corresponding pictures for the case where the source is moving.

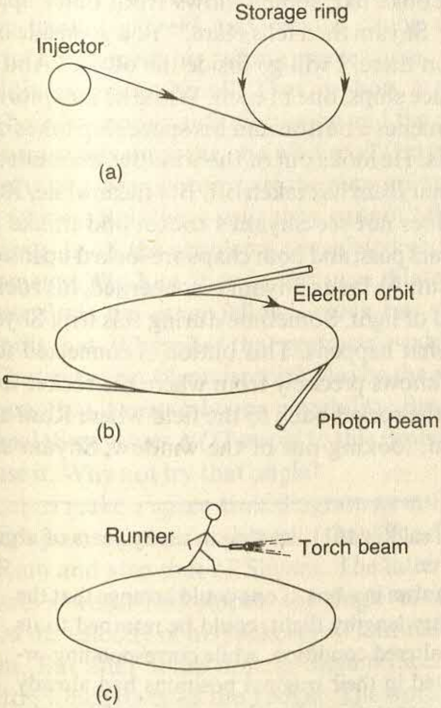


Fig. 7.4 Synchrotron radiation is produced by first accelerating electrons to a speed close to *c*. This is done in an accelerator called the injector. These electrons are then injected into a machine called the storage ring (see (a)) wherein the electrons go round and round in a circular orbit. However, since the electrons in the ring have a very high velocity, the radiation coming from them (called synchrotron radiation), experiences the "headlight effect". The beam is thus emitted tangentially and in a highly focussed manner as shown in (b). To get a simple physical picture, imagine the circulating electron pulse to be a runner going round a circular track and carrying a powerful torch as in (c). As the runner goes round, the light beam sweeps around. This is exactly what happens in a storage ring.

synchrotron radiation sources all over the world. Two such machines named INDUS 1 and INDUS 2 are under construction at the Centre for Advanced Technology in Indore.

8 *Some Paradoxes*

Relativity is full of "paradoxes", and it is time to discuss a few of them.

8.1 The twin paradox

We start with the twin paradox which goes back to a remark made by Einstein himself in his 1905 paper (see also Box 8.1). Let me state that in simpler terms.

Ram and Shyam are twins. One fine day when they are out in the open, they see two space ships. "Looks like some fellows from outer space have landed here," says Ram. Shyam then tells Ram, "You go inside one ship and look what it is like in there. I will go inside the other." And so they both get into the two space ships, one in each. While he is exploring inside, Shyam accidentally touches a button and his space ship takes off; but Shyam does not know this. He looks out of the window, does not see the other ship and so thinks that Ram has taken off. But meanwhile, Ram looking out of his window does not see Shyam's rocket and thinks his twin brother has taken off. Years pass, and both chaps are locked up inside; they don't know how to get out. As far as Shyam is concerned, his rocket is travelling close to the speed of light. Sometime during this trip, Shyam presses a button just to see what happens. This button is connected to a computer, and the computer knows precisely from where the rocket took off originally. It then guides the rocket back to the field where Ram and Shyam first saw it. And then, looking out of the window, Shyam sees

Box 8.1 This is what Einstein said in 1911, leading to many years of argument:

> If we placed a living organism in a box ... one could arrange that the organism, after any arbitrary lengthy flight, could be returned to its original spot in a scarcely altered condition, while corresponding organisms which had remained in their original positions had already long since given way to new generations. For the moving organism the lengthy time of the journey was a mere instant, provided the motion took place at approximately the speed of light.

Ram's rocket and tells himself, "So Ram is back. I wonder how his trip was." Ram also sees Shyam's rocket and thinks that Shyam is back. They now manage to find the handle that opens the door of the rocket, operate the handle to get out and then rush to see each other. Ram sees a young-looking Shyam, while he himself has grown quite old during the period he was trapped in the space ship. He then thinks for a moment and says, "Aha, of course Shyam must be young on account of relativistic time dilatation."

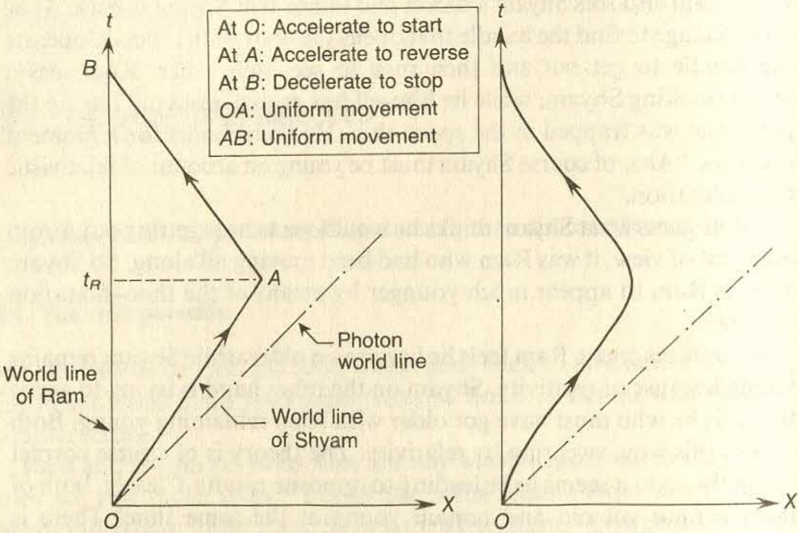
Let us guess what Shyam thinks he would see as he is getting out. From his point of view, it was Ram who had been moving all along. So Shyam expects Ram to appear much younger by means of the time-dilatation argument.

Now this is crazy. Ram feels he has grown older while Shyam remains young because of relativity. Shyam on the other hand is trying to argue that it is he who must have got older with Ram remaining young. Both reason this way, swearing by relativity. The theory is of course correct but in this case it seems to be leading to opposite results. Clearly, both of them cannot get old and remain young at the same time! There is obviously something wrong somewhere. Is the Special Theory of Relativity wrong after all? That, in short, is the PROBLEM!

There has been much discussion and the answer is that it is Ram who ages since Shyam is the one who *really* travels. You might say, "Listen, in relativity we only compare the view points of two chaps in two different frames. Relativity says they cannot tell among themselves who is moving. In all the problems considered so far, there has always been a symmetry. We had that in the time dilatation example earlier. Each fellow finds the other fellow's clock has slowed down, and relativity permits that. Where has that symmetry suddenly disappeared now?"

That *is* the problem, isn't it? Maybe the special theory of relativity is wrong; that is certainly one possibility. But another solution is that the special theory does *NOT* apply to this problem in the sense we are trying to use it. Why not try that angle?

Let us make a space-time diagram as in Fig. 8.1 which is drawn from Ram's point of view; no harm in that. The diagram shows the world line of Ram and also that of Shyam. The latter needs a bit of explanation. Shyam's world line slopes, the angle of inclination being dependent upon the velocity of his rocket ($v \sim c$). Initially, Shyam moves away from Ram. But after some time t_R Shyam (accidentally) hits the computer button which reverses the rocket. The world line thus bends over. I have oversimplified it in Fig. 8.1(a); in reality, the world line would really look more like that in Fig. 8.1(b) due to gradual take off and landing. The important feature is the overall *change in direction* of the world line.



(a) (b)

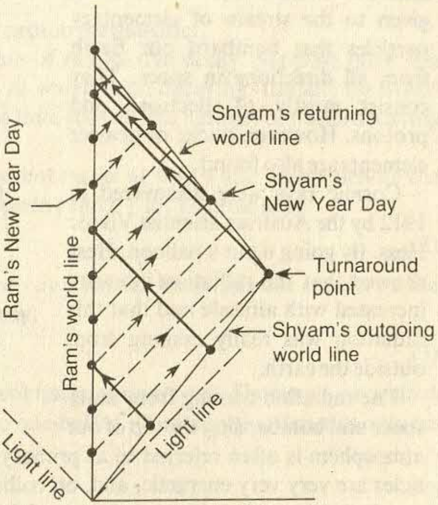
Fig. 8.1 World lines for Ram and Shyam drawn with reference to Ram's frame. Since Ram does not move, his world line is the same as the t-axis. (For convenience, we assume Ram is at the origin.) In (a) Shyam's trajectory has been idealized while in (b) a more realistic version is shown.

What does all this imply? Remember the conditions under which we are allowed to use the special theory. The two chaps must each be in an *inertial* frame. Even if we take an ideal scenario as in Fig. 8.1(a), Shyam has to accelerate when he reverses. So his is not an inertial frame, at least during the period of acceleration, even if the reversal took but just one second. During that period there would have been such a *terrific* acceleration that Shyam would have known that he is moving. Once he knows he is moving, he is no longer in an inertial frame, and we have no business applying the special theory from his point of view—it is illegal! So, the paradox appears because we are trying to sqeeze results using the special theory in an incorrect manner. Then what do we do? The symmetry that normally exists between two frames no longer applies here. However, all is not lost; it only means that while Ram can invoke special relativity and say Shyam is ageing the latter cannot do likewise.

According to Fig. 8.1(a), Shyam definitely experiences strong acceleration during take off, landing and while reversing. To a first approximation, Ram could ignore these three instants, and examine what happens during the part *OA* of Shyam's journey. During this period, Shyam is coasting uniformly, and Ram could safely apply special relativity to argue that Shyam remains young. Same during the part *AB* of Shyam's trip. Ram's ignoring the three brief periods of acceleration/deceleration does not alter the basic conclusion namely, that Ram ages while Shyam remains young.

The twin paradox is often made even more interesting by talking about the greetings exchanged by the twins every New Year Day. You will find a brief discussion of that in Box 8.2.

Box 8.2 You may call this the "greetings paradox". We have Ram and Shyam again, and we suppose that Shyam takes off on his long journey on New Year's Day. Before separating, both Ram and Shyam agree to exchange New Year greetings. And so, on January 1 every year, Ram dutifully radios a greeting to his twin brother Shyam. Shyam does likewise. Neither of them miss a year, until they get together. Though both send greetings on every New Year's Day, it turns out that Ram sends more greetings to Shyam than Shyam sends to Ram. How is this possible? The answer is illustrated in figure (a).



(a)

8.2 The muon paradox

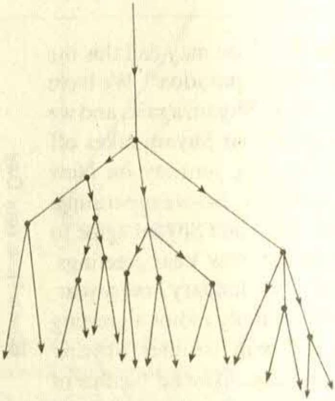
The *muon* (historically called the μ-meson) is a particle like the electron but about two hundred times more massive. Unlike the electron and the proton, it is not stable. When born, it dies in a fraction of a second by radioactive decay. Production of muons in the laboratory is not easy because that requires a particle accelerator. However, people with such facilities have not only produced muons, but also made them to replace electrons in atoms, to make what are called muonic or μ-*mesic atoms*.

It turns out that muons are also produced by Nature all the time. Our Earth is being continuously bombarded by particles from outer space called cosmic rays (see Box 8.3). As they enter the atmosphere, these cosmic rays produce a shower of muons which then travel downwards at speeds close to c. And that is what makes things interesting for us, because they exhibit a paradoxical behaviour.

The decay of the muons is described by the equation

$$\mu^- \to e + \nu + \tilde{\nu}, \tag{8.1}$$

i.e., a negatively charged μ-meson decays into an electron, a neutrino ν and an anti-neutrino $\tilde{\nu}$. (See Box 8.4 for a primer on radioactivity.) The


(a)

Box 8.3 Cosmic rays is the name given to the stream of elementary particles that bombard our Earth from all directions in space. They consist mostly of electrons and protons. However, nuclei of heavier elements are also found.

Cosmic rays were discovered in 1912 by the Austrian scientist Viktor Hess. By going up in a balloon, Hess showed that the radiation intensity increased with altitude and that this radiation was really coming from outside the earth.

The radiation coming from outer space and bombarding the top of our atmosphere is often referred to as primary cosmic rays. The incoming particles are very very energetic, and, on collision with the nuclei of the upper atmosphere, produce many other particles which then travel towards the earth. These are called secondary cosmic rays.

Figure (a) shows that a single incoming particle (a part of the primary radiation) can produce a shower by a cascade process. The study of such showers is intensely pursued by researchers all over the world, and here in India, such studies were pioneered by Homi Bhabha. Bhabha also gave a theory for the showers. Later, Vikram Sarabhai was active in the study of cosmic rays. The Tata Institute of Fundamental Research has been very active in the study of cosmic rays for nearly forty years. It has carried out such studies at high altitudes (using balloons), in mines and in lakes.

Two particles of present interest are the pion and the muon. The pion is about 240 times as massive as a proton. It can be positively charged (π^+), negatively charged (π^-) or neutral (π^0). The pion is short lived and in a little over a billionth of a second, it decays into a muon. The muons also disintegrate. The paradox discussed in the text arises from the fact that the *cosmic ray produced muons* travel at very high speeds and therefore exhibit relativistic effects.

positive muon decays in a similar manner, except that instead of the electron, a positron is produced. The neutrino is a tiny particle, rather elusive. It has no charge, and has practically zero mass. It not only plays an important role in radioactivity, but in cosmology as well.

Box 8.4 Radioactivity is the process whereby a nucleus spontaneously transforms or decays into another.

The simplest example is the decay of the neutron:

$$\text{Neutron} \rightarrow \text{proton} + \text{electron} + \text{neutrino}. \tag{1}$$

Here, neutron is the parent, and proton the daughter.

There is a rule giving the rate of radioactive decay. Suppose there is a collection of parent nuclei. Let us watch them decaying. Initially we would find many nuclei decaying but as time goes on, the number of nuclei decaying per unit time will go down.

Let $N(t)$ be the number of parent nuclei at time t and let dN denote the number decaying in the time between t and $t + dt$. The rule says

$$dN = -\lambda N(t)dt \tag{2}$$

where λ is a constant called the *decay constant*. From this one can derive the following law for population decay:

$$N(t) = N(0)e^{-\lambda t} \tag{3}$$

where $N(0)$ is the number of parent nuclei at time $t = 0$. The time $t_{1/2}$ required for the population to decrease to one-half of its initial value depends only on λ and not on $N(0)$. It is given by

$$t_{1/2} = \log_e(2)/\lambda = 0.693/\lambda. \tag{4}$$

The time duration $t_{1/2}$ is called the half-life of the parent element.

The half-life for the decay of the muon has been measured in the lab and found to be 2.2×10^{-6} sec or 2.2 micro sec (μ sec). Now radioactivity furnishes a clock in the following sense. Let us say a fire is burning. I don't have a stopwatch or even a watch to see how long the fire actually burns. However, I have a bunch of radioactive atoms. I count the number of these atoms at the start of the fire and I determine the number of such atoms present when the fire just dies out. In between, many atoms would have decayed. So I know the number of atoms I started with, and I also know the number of atoms I am left with at time t when the fire goes out. If I also know the half-life for the radioactive decay of the atom concerned, then, using the well-known equation

$$N(0) = N(t)e^{-\lambda t}, \tag{8.2}$$

I can say how long the fire lasted. In this equation, I know $N(0)$ the number of atoms I had to start with, and I also know the number $N(t)$ of atoms left at time t when the fire died. In addition, I know the decay constant λ. So I can find the time t for which the fire was burning. True, radioactivity gives only a *statistical* clock and is therefore not very accurate. But it suffices on many occasions. Since the muons are radioactive, they literally carry such a clock.

For the sake of simplicity let us say that the muons are all produced at a height of 7 km and they all have the same velocity v (which is close to c, of course). Instead of going 7 km up, let us climb a mountain in say Ooty or Kodaikanal in the south or in Mussoorie or Darjeeling in the north. We carry with us a detector of muons and start counting the muons. We take counts, say, every five minutes. Let us say we get numbers like 507, 554, 579, 602, 499, 547, ... etc. These numbers are not the same because the arrival of muons at our detector is a *statistical* or random process. Therefore, there *should* be fluctuations.

Even though the muon counts fluctuate, we can calculate an average of these by taking a large number of readings. Say the average is 543. We now come down the mountain, bringing our detector with us. We next go to a coastal station like Madras or Trivandrum or Bombay or Calcutta. Before getting off the mountain, we are careful to note its height. At the coastal station, we again take counts using our detector.

Even before we start counting, we can make a prediction of how many counts we should expect. How do we do this? We know the height H above sea level of the mountain we climbed. And now the second set of measurements are being done at the sea level. So, in the second experiment, the muons would have to travel an additional distance H. Since their velocity is v, they would take a time $t = (H/v)$ sec for this. Therefore, if $N(H)$ is the number of mesons found at a height H (for us this number would equal 543), in time $t = (H/v)$, the number of surviving muons would be $N(H) \cdot e^{-\lambda H/v}$. We should therefore expect to detect only this many number of muons at the sea level (on the average of course). When the experiment is actually done, one finds a number larger by several orders of magnitude.

Typical numbers taken from an actual experiment by Frisch and Smith are given in Table 8.1. Some details of their experiment are described in the Appendix. At the top of a mountain roughly 2000 m high, they detected, on the average, 563 per hour. At the sea coast they expected, using a calculation as I have described above (taking $t_{1/2} = 2.2\mu$ sec), to detect only about 27 mesons per hour. Instead, they systematically found over four hundred. This is the paradox. Why so

Table 8.1 Muon decay counts at the top of Mt. Washington (height ~2000 m) and at Cambridge, Massachusetts (sea level). Each run is for one hour.

Run	Mt. Washington	Cambridge
1	568	412
2	554	403
3	582	436
4	527	395
5	588	393
6	559	—
Average hourly rate	563 ± 10	408 ± 9

$t_{1/2} = 2.2\ \mu\text{ sec}$ $\lambda = (0.693/2.2\ \mu\text{ sec})$

Time for muons to come down by 2000 m is approx 6.4 μ sec. Therefore

$$563 \times e^{-(0.693/2.2)\times 6.4} \sim 27 \text{ counts/hour.}$$

many mesons in excess? The experiment was clean and there was nothing to doubt there. The value of 2.2 μ sec used for the half-life has been checked again and again, and there is nothing to doubt there either.

The answer lies in time dilatation. When we say the muons take a time H/v to travel a distance H, we are perfectly correct, but with respect to *our own frame*. If you ask the mesons how long *they* took to come down from the mountain top, they would look at *their* clock; but their's is a *moving* clock. So, what to us is a time H/v, would be much less for the muons. And during this time, naturally fewer muons would have decayed. Therefore, at the sea level, there must be a much larger number of survivors, as in fact one actually sees. In short, thanks to dilatation, the travelling clock shows that only 0.7 μ sec have elapsed whereas our clock shows 6.4 μ sec. The muons are moving while they are decaying and they have to go only by *their* clock and not by what our clock says.

8.3 The pole vaulter's paradox

This is an interesting problem. There is a barn—it is something like our cattle shed but modified for a cold climate and so there are walls and doors—whose length is, say, L_0. We shall refer to the doors in the barn as the front and the back door respectively—see Fig. 8.2. A pole vaulter whom we shall call A has a pole of length L_0. The doors of the barn are open, and the pole vaulter wants to run through the barn carrying with him the pole which he holds horizontally.

This A is no ordinary chap, for he can run at speeds close to c. (Don't

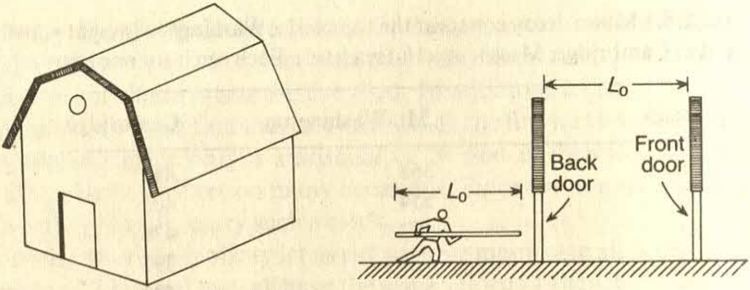


Fig. 8.2 This shows a runner with a pole of length L_0 trying to run through a barn with two doors. The length of the barn is also L_0.

ask me what vitamins he takes!) Now there is another person B inside the barn. This B does not like A. B is smart and knows his relativity. So he tells himself, "When this chap A is running, his pole will shrink in length due to Lorentz contraction. So after he gets into the barn, I shall arrange to close both the doors and trap him inside."

A is no fool either. He knows that B is trying to capture him inside, and he tells himself, "As far as I am concerned, relativity tells me the barn's length would shrink. Therefore, my pole would be *longer* than the barn. So how on earth can B ever trap me?" This is interesting. B says he can trap A inside on account of the Lorentz contraction of the pole, while A thinks he can escape because of the Lorentz contraction of the barn. Both are banking on the Lorentz contraction; and what each one wants is exactly the opposite! How can relativity please both? This is the question.

First let us consider B, the chap inside the barn. He is certainly correct in thinking that the pole of A would contract—see Fig. 8.3. In fact, for B, the length of the pole would be

$$L = L_0 \cdot (1 - v^2/c^2)^{1/2}. \tag{8.3}$$

Obviously, L is less than L_0, the difference being

$$\Delta L = L_0 - L. \tag{8.4}$$

Let us now look at things in A's inertial frame. Here the pole is stationary and the barn moves. In fact, it rushes towards A with a speed v. And so for A, it is the barn which shrinks—see Fig. 8.3. Therefore, A is certainly justified in thinking he can run through the barn without being trapped. In fact, for A the shrinkage is given by

$$\text{Length of pole} - \text{length of barn} = L_0 - L \tag{8.5}$$

where L is again given by (8.3).

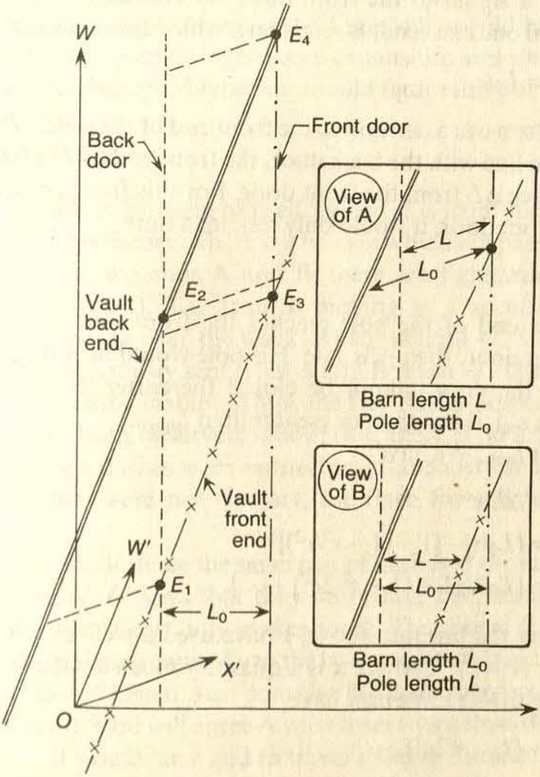


Fig. 8.3 Space-time diagram for the pole vaulter's problem. The W'-axis is the world line for A. The contraction from the separate points of view of A and of B are shown as insets. Of interest are the following four events: (i) E_1 or the front end of the pole coming into line with the back door, (ii) E_2 or the back of the pole coming into line with the back door, (iii) E_3 or the front of the pole coming into line with the front door, and (iv) E_4 or the back of the pole coming into line with the front door. After E_4, the pole is clear off the barn.

So far, I have merely repeated what I said earlier, though with a few formulae thrown in. I now come to the explanation. First let me tell you that B is the fellow who is fooled and that A escapes. B is certainly correct in thinking that the pole is shrunk. You might wonder: "How on earth can A then escape? If the pole is shrunk, surely A can be trapped." That is where the whole point is.

Let us join B and try to see what is going on inside the barn. B is hiding near the back door, and he watches A enter with his pole. A then advances through the barn, and some time later, the back end of the pole is in line with the back door—see Fig. 8.3; B closes the back door instantly. He now has to close the front door also if he wants to trap A.

So B sends a signal to the front door for closing it. Since light is the fastest signal one can send, **B** sends one, which takes the time

$$t_{\text{signal}} = L_0/c. \tag{8.6}$$

We now turn our attention to the front end of the pole. When the back end is just in line with the back door, the front end (in B's frame) is just a short distance ΔL from the front door. For this front end of the pole to reach the front door, it would only require a time

$$t_{\text{pole}} = \Delta L/v. \tag{8.7}$$

If the front end of the pole reaches the front door *before* B's signal reaches that door, then A is safe. His pole would be jutting through the door, and the door cannot be closed thereafter. So we have to ask ourselves if t_{pole} is less than or greater than t_{signal}.

Consider t_{pole}. We have

$$\begin{aligned} t_{\text{pole}} &= \Delta L/v \\ &= (L_0/v) \cdot [1 - (1 - v^2/c^2)]^{1/2} \\ &= (L_0/v)[1 - (1 - v^2/2c^2) + \ldots]. \end{aligned} \tag{8.8}$$

In writing the last line above, I have used a well-known formula for expanding $(1 - x)^{1/2}$, where x is a quantity much smaller than 1. Let us continue with (8.8). We now have

$$t_{\text{pole}} \sim (L_0 \cdot v/2c^2). \tag{8.9}$$

We must compare this with t_{signal} and show that t_{pole} is smaller, i.e.,

$$t_{\text{pole}} < t_{\text{signal}} \tag{8.10}$$

or,

$$L_0 \cdot v/2c^2 < L_0/c \tag{8.11}$$

or,

$$v^2/2c^2 < v/c \tag{8.12}$$

Now v/c is a number less than 1; so clearly $(v/c)^2$ will be smaller than (v/c) itself, in which case, $0.5(v/c)^2$ would be even smaller. What does it all mean? Simply that the *front end of A's pole would reach the front door before B's signal reaches that door*. And once the front end of the pole clears that door, there is no way in which B can trap A. Observe that though both A and B are in inertial frames, there is an asymmetry in their expectations as well as what happens eventually.

You might be worried that I have used the expansion in (8.8) in a shady manner. I have as a matter of fact, because normally I am supposed to use it only when x is much less than 1 whereas here x is close to 1. Fortunately, this is not a serious objection. All that one would have to do is include more and more terms in the series expansion, and when one does that, one does even better. Maybe you should figure this out yourself!

8.4 The train paradox

This one goes back to Einstein himself. In fact it is from such problems that he was led to his theory. There is a train in which is a passenger O' at the middle. At the two ends A and B, there are lights which can be flashed—see Fig. 8.4(a). The train is moving at a speed close to c. Observer O is standing near the track. At the instant O' is crossing O, two light flashes previously sent from A and B reach O'. O also sees the two flashes at the same instant. Thus, the two flashes appear to arrive simultaneously to both observers; about this, there is no argument. O' now says, "The two flashes were *emitted* simultaneously in the past." O says, "No sir, they were not. In fact, the flash from B was emitted earlier."

This is really odd. Both see the same pair of flashes at the same instant, i.e., simultaneously. O' says that they must have been emitted earlier simultaneously, which certainly makes sense. Therefore, O appears to be wrong. But if we listen carefully to the latter we would find that he has a point too. O says, "Listen. Just consider the instant before O' crossed me (see Fig. 8.4(b)). You will agree A was closer to me than B. Thus light from the source B would have had to travel a *longer* distance in order to reach me. If light from both the sources reached me at the same instant,

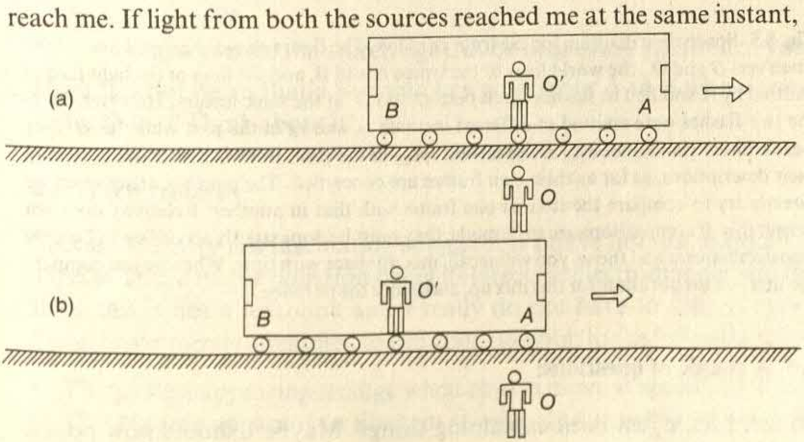


Fig. 8.4 (a) This shows a train carrying an observer O' (at the middle of the train), and two flash lamps A and B at the two ends of the train. The lamps are flashed in such a manner that when O' crosses O, both O and O' see the two flashes arrive simultaneously. (b) shows the scenario moments before O' crosses O. Observe that B is farther from O than A is.

clearly the light flash from B must have left *before* that from A." When he puts it like this, certainly *O* seems correct.

We now have a problem. Who is right, *O* or *O′*? The arguments of both sound convincing but surely both can't be right! Which one is it?

Special theory resolves the difficulty by saying both are correct! That seems hard to swallow, does it not? It should not be if you consult Fig. 8.5. The train paradox is a particular case of the relativity of simultaneity, earlier discussed in section 5.13.

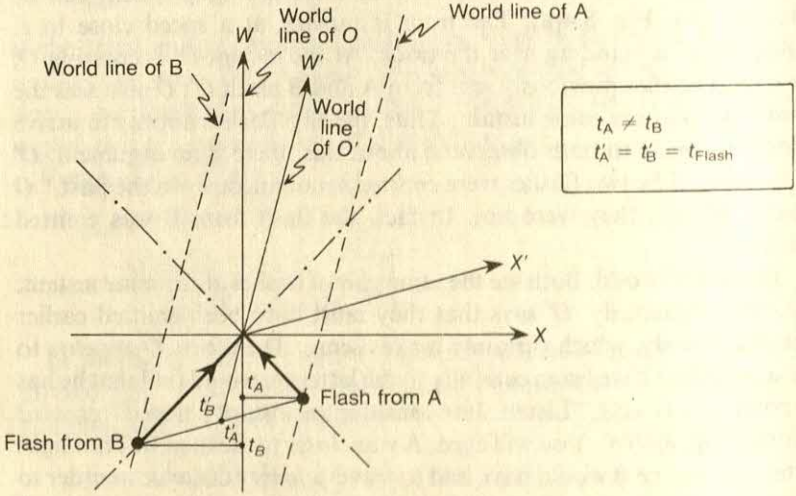


Fig. 8.5 Space-time diagram for the train paradox. The figure shows the world lines of the observers *O* and *O′*, the world lines of the lamps A and B, and the lines of the light flashes emitted by A and B. The flashes reach both *O* and *O′* at the same instant. However, for *O* the two flashes were emitted at different instants t_A and t_B in the past while for *O′* they were emitted simultaneously at the instant t_{Flash} in the past. Both *O* and *O′* are correct in their descriptions, as far as their own frames are concerned. The paradox arises when we directly try to compare the time in one frame with that in another. Relativity does not permit this. If comparisons are to be made, they must be done strictly according to Lorentz transformations; and those, you will recall, mix up space with time. When we just compare the time, we do not allow for this mix up, and hence the paradox.

8.5 A couple of questions:

So far, I have just been explaining things. Maybe I should now pose a couple of questions!

(a) Here is one which Einstein is reported to have thought about when he was a boy. A person runs at the speed of light (—believe, for a moment, that is possible!) holding a mirror in front of him—see Fig. 8.6. Will the runner be able to see himself in the mirror?

(b) A powerful light is rapidly rotated, like in a lighthouse. A and B are standing some distance away and watching the beam as it sweeps past. Both are at the same distance R away from the source—see Fig. 8.7. R is so large that the beam sweeps from A to B faster than a light signal can travel from A to B.

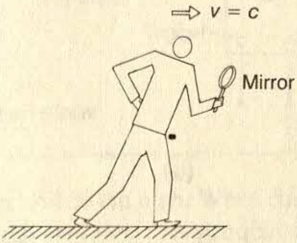


Fig. 8.6 This illustrates a runner carrying a mirror. The runner is running at a speed equal to c. The question is: Can he see himself in the mirror?

Before they take up positions, A and B are given the following instructions:

Instruction to A:

When you see the search light, fire a bullet at B.

Instruction to B:

When you see the search light, duck because A has fired at you.

Does this not mean that a warning has gone from A to B with a speed faster than c? Think about it!

8.6 A few remarks

Special relativity is a fascinating subject and I have not discussed all its aspects here, a major omission being its effects in electromagnetism. But then, this is not a textbook and I really do not have to discuss everything. I have merely attempted to call your attention to the following facts:

- Things start appearing strange when objects move at speeds close to c.
- This became apparent to Einstein at an early age, when he posed to himself many awkward questions.
- While searching for answers, Einstein stuck to two cardinal rules which we know as the two postulates.
- Eventually, Einstein came up with the Lorentz transformations and

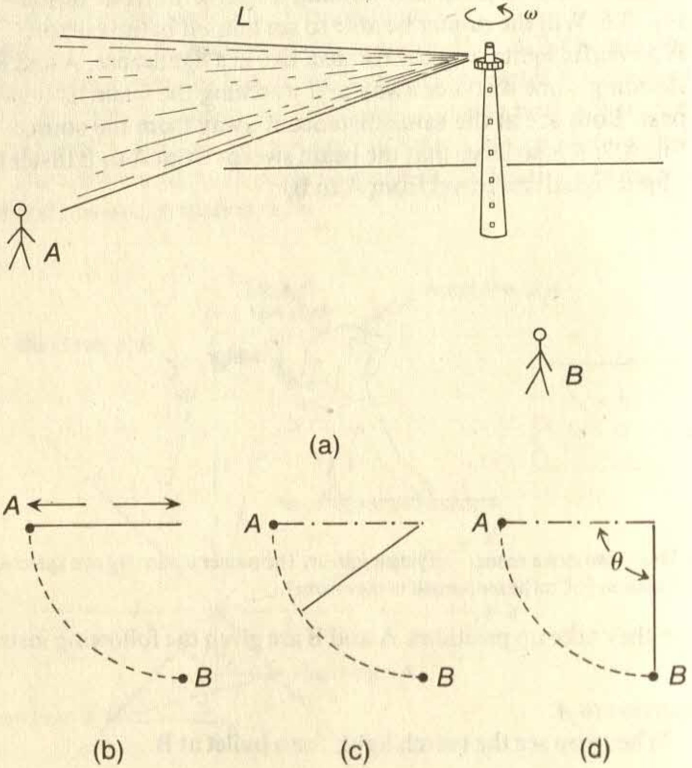


Fig. 8.7 *L* is a beam of light, rotating at an angular speed of ω degrees/sec. A and B are two observers located at a large distance *R* from the source. The sketches in (b), (c) and (d) depict the situation at three instants as the beam sweeps around. If ω is large, the time (θ/ω) can be made smaller than the time required for a light signal to be sent from A to B. With this information, you should be able to tackle the problem.

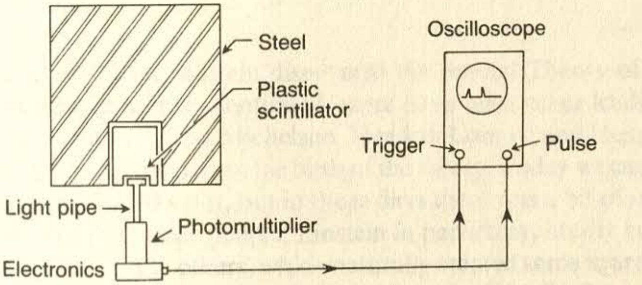
with them showed how both mechanics and electromagnetism ceased having contradictions.

Of course, this meant modifying Newtonian mechanics suitably. But the modifications were such that Newtonian mechanics is still OK when velocities are small.

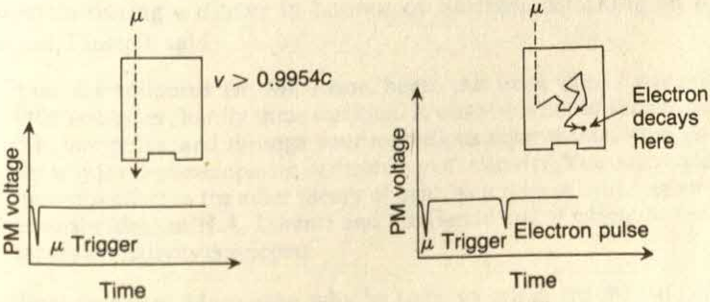
You might have noticed that I have always been harping on inertial frames. You might wonder what happens in accelerated frames. A brief glimpse of that is given in the last chapter.

Appendix to Chapter 8

Given here is some idea of the counting equipment used by Frisch and Smith in their experiments. Basically, the particles are detected using a scintillation counter—see (a). This consists of a big cylinder of plastic, roughly 30 cm in diameter and 30 cm high. When charged particles travel through this at high enough energies, they produce a light flash, which is picked up by a photomultiplier. The multiplier gives an electrical pulse as an output which can be seen using an oscilloscope. The detector is surrounded by a big iron assembly which completely stops mesons with velocity less than 0.995 *c*. If the velocity of the meson is greater than 0.995 *c*, it passes through the iron into the scintillator. Now if the velocity is greater than 0.9954 *c*, the meson shoots through the scintillator too and escapes. But if its velocity is between 0.9954 *c* and 0.995 *c*, then it is unable to escape from the scintillator, meaning that while it is travelling through the plastic cylinder it decays according to equation (8.1). Whenever a meson hits the scintillator, the oscilloscope trace is triggered. At the same time, the photomultiplier pulse is also fed to the deflection plates of the scope and shows up as either in b(i) or b(ii). In



(a)



b(i) b(ii)

the latter, one sees a second pulse due to the decay of the muon in the scintillator. We also know that the velocity of the decaying meson is between 0.9954 *c* and 0.995 *c*. Smith and Frisch counted how many events like those in b(ii) occurred during an hour, when the detector was on Mt. Washington and then at Cambridge, Massachusetts.

9 *A Bit Of History*

It is known that Einstein discovered the Special Theory of Relativity. However, as earlier mentioned, there have been other leading personalities namely, Albert Michelson, Hendrik Lorentz and Henri Poincaré, who also played a part in the birth of the theory. Today we can clearly say who contributed what, but in those days there was a bit of uncertainty. Added to this, some people, Einstein in particular, hardly ever referred to the work of the others, which naturally created some heartburns.

Let us start with Einstein and Michelson. In 1950, Professor Shankland met Einstein and they discussed Michelson's work. Einstein told Shankland that he became aware of the Michelson–Morley result only *after* he had written his paper on relativity in 1905. Then how did Einstein know that motion through ether did not affect light propagation? Einstein replied that it was from Fizeau's experiment. Much later Einstein wrote, "In my own development, Michelson's result has not had a considerable influence." It is curious that Einstein hardly ever referred to Michelson's experiment in later years, even after he became aware of it.

Michelson won the Nobel Prize in 1907. He was the first American to do so. In his acceptance lecture, Michelson made no reference to Einstein's theory published two years earlier. One could perhaps excuse Michelson since Einstein's theory was still new and had not yet become well-known. Possibly, Michelson was not yet aware of it.

Michelson and Einstein met only once, and that was in 1931 in America during a dinner in honour of Einstein. Speaking on the occasion, Einstein said,

> You, my honoured Dr. Michelson, began this work when I was only a little youngster, hardly three feet high. It was you who led the physicists into new paths, and through your marvellous experimental work paved the way for the development of the theory of relativity. You uncovered an insidious effect in the ether theory of light, as it then existed, and stimulated the ideas of H.A. Lorentz and FitzGerald out of which the special theory of relativity developed.

Einstein asked Michelson why he took so much trouble to do those experiments. Michelson replied, "Because it was fun."

Years later on the occasion of the centenary of Michelson, Einstein sent a message in which he said:

> I always think of Michelson as the artist in Science. His greatest joy seemed to come from the beauty of the experiment itself, and the elegance of the method employed. But he has also shown an extraordinary understanding of the baffling fundamental questions of physics. This is evident from the keen interest he has shown from the beginning for the problem of the dependence of light on motion.

Einstein told Shankland that he had started working on relativity from the time he was sixteen, spurred by many questions which occurred to him. As a student, he could not spare much time, and many of his attempts were a failure. The point is that Einstein was not bothered by Michelson's result (like Lorentz was, for example). He arrived at relativity entirely on his own.

Next we consider Lorentz. Unlike Einstein, Lorentz was very much aware of Michelson's null result. In fact, he wrote in 1892, "This experiment has been puzzling me for a long time, and in the end I have been able to think of only one means of reconciling it with Fresnel's theory." This was by introducing the Lorentz contraction. Meanwhile, Lorentz became aware that FitzGerald the Irish physicist had independently proposed the idea of contraction earlier, and so he wrote to FitzGerald. FitzFerald was very happy that Lorentz agreed with him. He was worried that people would think contraction to be a silly idea. In 1895, Lorentz published a paper in which he discussed the relationship between electric and magnetic fields in two inertial frames. The seeds for the famous Lorentz transformations were sown in this paper. And in 1899, he came even closer to writing down those transformations. In spite of all this, Lorentz did not come up with anything of major importance. In fact, even after his work, he still believed that there was an ether, that it was at rest, and that speeds greater than *c* were possible.

Einstein was aware of the work of Lorentz. He tried to push it further, but was dogged by problems. One day Einstein visited a friend to discuss a few things and to air his doubts and problems. Suddenly during the conversation Einstein had a flash. He left abruptly to ponder alone. The next day Einstein called on his friend again and said, "Thank you, I have completely solved the problem." He had figured out that the problem was with *time*. Once he knew this, it required only a few weeks to put everything together and come up with a proper theory. Einstein and Lorentz liked each other very much and got along fine.

Next we come to Poincaré. As I have mentioned in Box 9.1, Poincaré was basically a mathematician; but he also tackled many problems of physics which involved mathematics. In 1898, he wrote a paper entitled

Box 9.1 Henri Poincaré was born in France in 1854. Henri's father was a Professor of Medicine. As a young student, Henri attracted attention with his unusual mathematical abilities, and in fact his teacher called him a "monster of mathematics". For a short while after he got his degree he worked as an engineer, but his heart was in mathematics and so, even while he was working, he also did a thesis in mathematics in order to obtain a doctorate degree. And once he got that, he entered the academic world wherein he stayed till the end. Most of his life, he spent as Professor at the University of Paris.

Poincaré worked both on mathematics as well as physics but he was more of a mathematician than a physicist. In mathematics, he was guided by the French mathematician Hermite. Poincaré contributed to many branches of mathematics such as function theory, topology, etc. He was very strong in differential equations, and very seldom let a year pass without publishing a paper on the subject. Understandably, he went on to the study of problems of mechanics and celestial dynamics. Poincaré was also interested in electromagnetic theory, which then led him to study Maxwell's equations; and in the process he discovered the Lorentz transformations. As I have said in the text, it was Poincaré who named these transformations after Lorentz. Einstein also independently discovered the same set of transformations, which he then used to develop his theory of relativity.

On the Measurement of Time. In this he said, "we have no direct intuition about the equality of two time intervals. People who believe they have this intuition are the dupes of an illusion." In short, Poincaré was cautioning about the use of concepts like simultaneity. At that time, Poincaré was concerned generally about the ether rather than electromagnetic phenomenon. In a lecture delivered in 1904 in America, Poincaré said, "Perhaps we must construct a new mechanics, of which we can catch only a glimpse ... in which the velocity of light would become an unpassable limit ... We are not yet there." On June 5, 1905, Poincaré sent a short paper dealing with electrodynamics. This paper was really the summary of a longer paper which he later sent for publication in July. Meanwhile, without being aware of any of this, Einstein sent his paper on relativity for publication on June 30, 1905.

In his paper, Poincaré introduced Lorentz transformations as we know them today. Not only that, he said that he was naming them Lorentz transformations. He showed that Maxwell's equations remained covariant under the Lorentz transformation, that velocities add like in (6.2), and finally, that equation (5.11) also holds. Does that mean that Poincaré discovered the theory of relativity simultaneously with Einstein? Not quite, because in Poincaré's theory, there are, in addition to the two hypotheses which Einstein also makes, a third one. Poincaré says, "One needs to make a third hypothesis ... A body in translational motion suffers a deformation in the direction in which it is displaced." Poincaré repeated this as late as 1909. According to him therefore, Lorentz contraction is not a *consequence* of the first two postulates of relativity but an *independent* postulate that had to be built into the theory. As we know, this is not correct.

Poincaré and Lorentz appeared to get along well but between Einstein and Poincaré there was undoubtedly some tension. Poincaré never mentioned Einstein in his papers, nor did he refer to Einstein when he gave lectures on relativity in Germany in 1909. Poincaré and Einstein met only once, and that was in 1911. Einstein came away with the impression that though Poincaré was remarkably clever, he (Poincaré) did not believe in the Special Theory of Relativity! Finally, a couple of months before his death (in 1955), Einstein said, "Lorentz had already recognised that the transformations named after him are essential for the analysis of Maxwell's equations, and Poincaré deepened this insight still further ..."

In the message which he sent on the occasion of Michelson's centenary (to which I have already made a reference), Einstein also said,

> The influence of the crucial Michelson–Morley experiment upon my own efforts has been rather indirect. I learned of it through H.A. Lorentz's decisive investigation of the electrodynamics of moving bodies (1895) with which I was acquainted before developing the special theory of relativity. Lorentz's basic assumption of an ether at rest seemed to me not convincing in itself and also for the reason that it was leading to an interpretation of the result of the Michelson–Morley experiment which seemed to me artificial. What led me more or less directly to the special theory of relativity was the conviction that the electromotive force acting on a body in motion in a magnetic field was nothing but an electric field. But I was also guided by the result of the Fizeau-experiment and the phenomenon of aberration.

Sir Edmund Whittaker's name is well known as that of an applied mathematician. In 1910, he published a book called the *History of the Theories of Aether and Electricity*. This covered the period upto about the year 1900. In 1953, Whittaker published a second volume that covered the period 1900 to 1926. In this volume, Whittaker said that there was really nothing new about Einstein's theory and that it was just a minor extension of the work of Lorentz and of Poincaré. According to Whittaker, this is all that Einstein did but somehow his theory "attracted much attention". What is even more surprising is that when Einstein died in 1955, it was Whittaker who wrote the biographical memoir of Einstein for the Royal Society; and in that also he maintained that Einstein's theory was but a minor extension of the work of Lorentz and of Poincaré. A long time ago, Max Born had warned Whittaker against writing such nonsense but Whittaker did not pay any attention. It is not surprising therefore that a number of experts have severely criticised Whittaker's account of the history of the ether. Einstein's own reaction was: "I do not have to read the thing ... If he manages to convince others, that is their own affair."

10 *The Later Years*

10.1 Academic career

In Chapter 1 we left Einstein as a young man about to enter the academic world. Let us pick up his life-story from there. In 1909, Einstein was appointed a staff member in the University of Zurich. Meanwhile, the Karl–Ferdinand University in Prague, Czechoslovakia, was looking for a Professor of Physics. Shortly after recommending Einstein to the post, Max Planck the founder of quantum theory, said, "His work on relativity probably exceeds in audacity everything that has been achieved so far in speculative science and even in epistemology; non-Euclidean geometry is child's play by comparison." Thus it was in March 1911, that Einstein arrived in Prague to take up appointment as Professor. Einstein liked Zurich city better, but nevertheless, he seemed to enjoy working in Prague. "I have a splendid institute in which I work comfortably," he wrote to his friend and collaborator Grossman. To another friend Besso he wrote, "My position and my institute give me much joy." Still in late 1912, Einstein left Prague for Zurich. This time he served in the ETH where he was once a student. But the stay in Zurich was brief for Berlin now claimed him. This was a fateful move which later changed the course of his life.

Einstein moved to Berlin to serve as Professor in the University there. This was a prestigeous appointment for the job involved several other important duties. Meanwhile the First World War (1914–1918) intervened, and Einstein declared himself a pacifist.

As I mentioned in the first chapter, Einstein belonged to the Jewish faith. True he was not a very religious man but he felt close to the Jewish people because, despite their numerous contributions, Jews were always looked down upon in the various countries of Europe. Slowly the anti-Jewish feeling was building up. The Jews then organised themselves into a body called the Zionist movement, and Einstein became friendly with leading Zionists. Soon the Zionists started claiming a homeland for themselves, and it is out of this demand that the state of Israel was eventually born.

In 1920, disturbances broke out during the course of a lecture given by

Einstein at the University of Berlin. The trouble was created by people who hated Jews; Einstein became a target since he was now a famous man. Meanwhile, Germany was going through a very difficult period, having been defeated in the First World War. It became convenient for some Germans to hold the Jews responsible for all the problems of their country.

Things, however, were not yet as bad as they were to become a few years later. And so Einstein continued to stay in Berlin, although he was unhappy about attacks against the Jews and often against "Jewish science" too. Meanwhile, Einstein bought himself a nice little house by a lake so that he could go sailing.

In October 1922, Einstein went on a long trip to many countries including Sri Lanka (then called Ceylon), Singapore, China, Japan, Palestine and Spain. It was while he was on this trip, that he learnt that he had been awarded the Nobel Prize.

In 1924, S.N. Bose, inspired by Einstein's work on the photon, wrote a paper which he sent to Einstein for his opinion. Einstein quickly realised that Bose's work contained a very important idea and got the paper published in the *Zeitschrift fur Physik*. For further details of that story, see the companion volume *Bose and His Statistics*.

Einstein visited America in 1930–1931, as a Visiting Professor at the California Institute of Technology (popularly known as Caltech). By this time things had become quite bad for Jews in Germany. Adolf Hitler had seized power and he made no secret of his rabid hatred for the Jews. Everyone was quite afraid. Hitler began to viciously persecute the Jews. Those who could, fled, including many leading scientists. Things became so bad in Germany that people were told not to believe in anything discovered by Jewish scientists, and this included the theory of relativity!

Einstein was now determined to quit Berlin and Germany for ever. He returned to Germany (from America) for a brief while and in late 1932, he left for America again. As he locked up his Berlin house, his wife told him, "Turn around. You will never see it again." And so it was.

Einstein went back to Caltech, but only for a brief period. Later he moved to the Institute for Advanced Studies in Princeton, New Jersey. In 1935, Einstein bought a small house at 112, Mercer Street, Princeton, and that was his home till he died.

In 1939, the Second World War broke out. America did not enter the war straightaway, but its sympathies were with Britain, France and the other countries fighting Hitler. Meanwhile the phenomenon of nuclear fission had been discovered, and scientists started speculating whether one could build an atomic bomb. Europe had been practically runover by Hitler, and scientific activity shifted entirely to America. When the possibility of building an atomic bomb became known, scientists in Western Europe and America voluntarily stopped publishing their results on nuclear fission.

Building the atomic bomb is complicated business. Of course no one knew as yet *how* such a bomb should be built but everyone knew that it would take a lot of money and several hundred scientists to do the job. The only country which could afford such a costly project was America, and the question was how to interest the American Government in such a project. It was finally decided by the scientists that they would request Einstein to write a letter to the American President Mr. Roosevelt. Einstein was world famous, and a letter from him would carry weight. And so Einstein wrote to President Roosevelt telling him about the possibility of making the atomic bomb, and how it was important that America develop the weapon before Germany did (see Box 10.1). That turned the tide.

Box 10.1

Albert Einstein
Old Grove Rd.
Nassau Point
Peconic, Long Island

August 2nd, 1939

F.D. Roosevelt,
President of the United States
White House
Washington, D.C.

Sir:

Some recent work by E. Fermi and L. Szilard, which has been communicated to me in manuscript, leads me to expect that the element uranium may be turned into a new and important source of energy in the immediate future. Certain aspects of the situation which has arisen seem to call for watchfulness and, if necessary, quick action on the part of the Administration. I believe therefore that it is my duty to bring to your attention the following facts and recommendations:

In the course of the last four months it has been made probable—through the work of Joliot in France as well as Fermi and Szilard in America—that it may become possible to set up a nuclear chain reaction in a large mass of uranium, by which vast amounts of power and large quantities of new radium-like elements would be generated. Now it appears almost certain that this could be achieved in the immediate future.

This new phenomenon would also lead to the construction of bombs, and it is conceivable—though much less certain—that extremely powerful bombs of a new type may thus be constructed. A single bomb of this type, carried by boat and exploded in a port, might very well destroy the whole port together with some of the surrounding territory. However, such bombs might very well prove to be too heavy for transportation by air.

The United States has only very poor ores of uranium in moderate quantities. There is some good ore in Canada and the former Czechoslovakia, while the most important source of uranium is Belgian Congo.

In view of this situation you may think it desirable to have some permanent contact maintained between the Administration and the group of physicists working on chain-reactions in America. One possible way of achieving this might be for you to entrust with this task a person who has your confidence and who could perhaps serve in an unofficial capacity. His task might comprise the following:

a) to approach Government Departments, keep them informed of the further development, and put forward recommendations for Government action, giving particular attention to the problem of securing a supply of uranium ore for the United States;
b) to speed up the experimental work, which is at present being carried on within the limits of the budgets of University laboratories, by providing funds, if such funds be required, through his contacts with private persons who are willing to make contributions for this cause, and perhaps also by obtaining the co-operation of industrial laboratories which have the necessary equipment.

I understand that Germany has actually stopped the sale of uranium from the Czechoslovakian mines which she has taken over. That she should have taken such early action might perhaps be understood on the ground that the son of the German Under-Secretary of State, von Weizsacker, is attached to the Kaiser-Wilhelm-Institut in Berlin where some of the American work on uranium is now being repeated.

Yours very truly,

Albert Einstein

The American Government spent billions of dollars, and the atomic bomb became a reality. It was dropped not on Germany but on Japan. After the war, Einstein became opposed to nuclear weapons and often expressed regret about having written that fateful letter. As he once said, "Had I known that the Germans would not succeed in producing an atomic bomb, I would not have lifted a finger."

Einstein now began to speak strongly in favour of world peace. "The war is won but peace is not," he once declared. He very much believed in world government and strongly supported it. In those days there was what was called a *Cold War* between America and its allies on the one hand and the Soviet Union (now defunct) and its allies on the other. In America, talk about world peace was generally not tolerated and there was a Senator called McCarthy who accused people supporting world peace as the enemies of America! Most Americans were mortally afraid of McCarthy, and Einstein alone could get away because of his world stature. Einstein felt that the only way of fighting McCarthyism and the opposition to world peace was to launch a movement like the civil disobedience movement or *satyagraha* launched by Mahatma Gandhi in India during the thirties. As he once said, "It is my belief that the problem of bringing peace to the world on a supranational basis will be solved only by employing Gandhi's method on a large scale." Einstein was a great admirer of Mahatma Gandhi and of him once said: "Generations to come will scarce believe that such a one as this ever in flesh and blood walked upon this earth." He also had great respect for Jawaharlal Nehru (see Box 10.2).

At the Institute for Advanced Studies, Einstein worked alone, immersed in his search for a unified field theory, about which I shall say

Box 10.2

Princeton, New Jersey
U.S.A.

February 18, 1950

Dear Mr Nehru

I have read with extreme interest your marvellous book *The Discovery of India*. The first half of it is not easy reading for a Westerner. But it gives an understanding of the glorious intellectual and spiritual tradition of your great country. The analysis you have given in the second part of the book of the tragic influence and forced economic, moral and intellectual decline by the British rule and the vicious exploitation of the Indian people has deeply impressed me. My admiration for Gandhi's and your work for liberation through non-violence and non-cooperation has become even greater than it was already before. The inner struggle to conserve objective understanding despite the pressure of tyranny from the outside and the struggle against becoming inwardly a victim of resentment and hatred may well be unique in world history. I feel deeply grateful to you for having given to me your admirable work.

With my best wishes for your important and beneficent work and with kind greetings,

Yours cordially,

Albert Einstein

Please remember me kindly to your daughter.

something shortly. Nothing concrete emerged, but then Einstein was tackling a most difficult problem which not many would dare to touch. Although success eluded Einstein, everyone respected him—not only as a scientist but even more so as a man.

In this book we have been concerned mainly with the Special Theory of Relativity. But that is not Einstein's only contribution to science. (Even if it were, it is sufficient to win him a place in history.) Let me now mention his other major contributions which are: (i) his work on the "reality" of molecules, (ii) his contribution to the concept of the light quantum, and (iii) his *General Theory of Relativity*. Of the above, I shall skip item (ii) as I have discussed it to some extent in the companion volume *Bose and His Statistics*. We turn now to the remaining two.

10.2 Reality of molecules

Today we take atoms and molecules for granted but in the last century it was chemists who bothered more about atoms and molecules rather than physicists. In fact, physicists rarely invoked atoms till around 1850 or so. Mechanics, optics and thermodynamics were all developed without any concern about the fact that matter is made up of atoms. Some physicists did not even believe in atoms! More about that shortly.

One of the first attempts to bring atoms into physics was by Maxwell who developed a *kinetic theory* of gases. People knew about Boyle's law and Charles' law, and Maxwell decided to see if a gas imagined to be made up of moving atoms which collided when they came in each other's way obeyed the gas laws. And thus was born a *molecular theory* of gases. Ludwig Boltzmann of Austria carried these ideas further and showed that heat is nothing but the random motion of atoms in the substance, be it a solid, or a liquid, or a gas.

It is rather interesting that as late as the end of the last century, many leading physicists did not accept the existence of atoms. This included Ostwald (known for his law of diffusion) and Mach (who has been mentioned earlier in Chapter 5). In 1895, Ostwald publicly attacked the idea of molecules in a lecture he delivered to a scientific audience in Germany. Boltzmann opposed Ostwald and a lively debate followed. The audience, at least the mathematicians in it, supported Boltzmann. Even so Boltzmann felt deeply hurt by the attacks against him, and later committed suicide.

Let us get back to Einstein. One of the papers published by him in 1905 was entitled *On the Movement of Small Particles in a Stationary Liquid Demanded by the Molecular Theory of Heat*. This paper dealt with what would happen say, if a tiny dust particle was suspended in a beaker of water. Such an experiment was in fact performed in 1827 by a botanist called Robert Brown. He found that pollen grains suspended in water moved about in an irregular manner (see Fig. 10.1); this wild, dancing motion is referred to as *Brownian motion.* For a while it was thought that this irregular motion was due to the pollen grains having "life" in them but this theory was discarded when it was found that even particles of powdered rock and coal performed this wild dance. Other experiments showed that the suspended particles moved about in an irregular fashion because they were receiving random kicks from the molecules in the liquid. The higher the temperature of the liquid, the more vigorous was the Brownian motion. When Einstein wrote his paper, he was not aware of Brown's work but he dealt nevertheless with precisely the phenomenon which Brown had observed. It was only later that Einstein realised that he had actually given a theory for Brownian motion. This paper of Einstein's is a landmark in that it triggered a whole lot of other investigations. The French scientist Jean Perrin made detailed studies to verify Einstein's theory and was rewarded with the Nobel Prize.

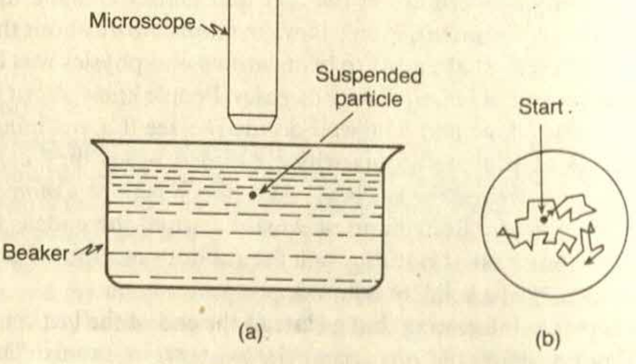


Fig. 10.1 (a) Schematic of the arrangement used by Robert Brown to observe the motion of suspended particles. (b) The Brownian particle essentially does a random walk. If the position of the particle after every one-second interval (say) is marked and these positions are joined by lines, we would get the trajectory of the random walk. Einstein was the first to analyse random walk quantitatively.

10.3 The General Theory

The General Theory of Relativity is a logical successor to the Special Theory. It is highly mathematical in nature and when proposed, few understood it. This was more due to the revolutionary ideas contained in the theory than due to its mathematical character.

It is not possible to discuss the general theory here; that would require a whole book. However, a few basic points must be mentioned. Consider two reference frames: (1) A nonaccelerating frame S in which there is a *uniform* gravitational field, and (2) a frame S' which is accelerating *uniformly* with respect to an inertial frame but in which there is no gravitational field. To fix our ideas, S could be a rocket stationary on Earth, and S' an identical rocket deep in space, far away from stars, etc. (see Fig. 10.2). Let S' accelerate with an uniform acceleration whose numerical value equals the acceleration due to gravity. Observers A and B in S and S' respectively, would have similar experiences. For example, when A drops a stone he would see it falling to the floor (of S). Likewise, when B drops a stone he too would see it falling to the floor (of S')—see Fig. 10.3.

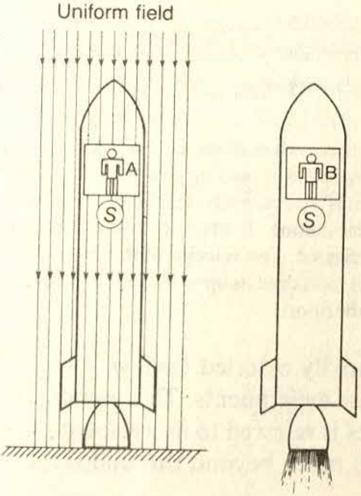


Fig. 10.2 Consider two reference frames S and S'. S is stationary but in a uniform gravitational field, while S' is *uniformly* accelerating but in a region totally free from gravitational fields. The principle of equivalence states that observers in the frames cannot say whether they are experiencing a uniform gravitational field or are being accelerated uniformly, i.e., S and S' are completely equivalent.

Let us now compare S and S'. The former does not accelerate but experiences a (uniform) gravitational field whereas S' accelerates (uniformly) but does not experience a gravitational field. Einstein declared that there is really no difference between S and S' and this is known as the *principle of equivalence*. What it means is that if rocket S and S' did not have any windows, A and B cannot say whether they are uniformly accelerating or experiencing a uniform gravitational field.

In a manner of speaking, Newton himself recognised the principle of equivalence by reasoning as follows: When a force F is applied to a mass, it accelerates. The formula connecting the two is $F = ma$; here, m is the *inertial* mass. To identify it as such, we shall write m_I instead of m.

Gravity acts on mass and in the case of our Earth this produces a force mg. The mass m entering the picture here is the *gravitational* mass denoted by m_G say. Thus, given a sphere of iron, we can attribute to it

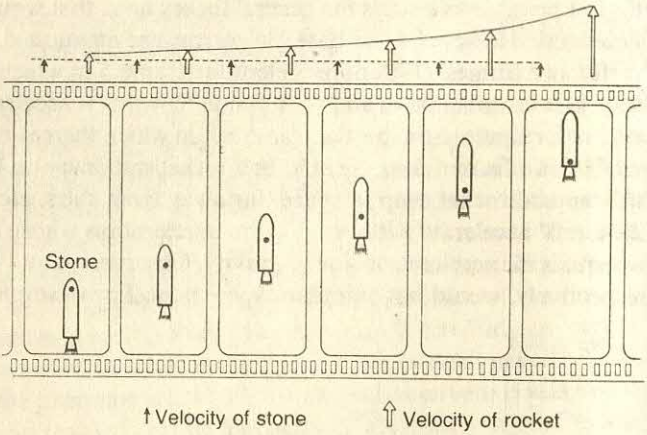


Fig. 10.3 Motion picture of the events that occur when observer B in rocket S' drops a stone. Originally both the stone and the rocket are moving with the same velocity. However, the moment the stone is released, it is mechanically decoupled from the rocket and therefore no longer experiences any acceleration. It obeys Newton's first law and moves with whatever velocity it had when released. The velocity of the rocket however, keeps on increasing. So the floor of the rocket soon catches up with the stone. This is what makes B think that the stone has dropped to the floor.

two masses m_I and m_G. Newton boldly asserted that $m_I = m_G$, a fact which has been confirmed by precise experiments. This equality of the inertial and the gravitational masses is referred to as *weak* equivalence. Einstein's statement obviously goes much beyond this and is therefore referred to as *strong* equivalence.

The principle of strong equivalence is the starting point of the general theory of relativity. Einstein has stated that when he got the idea for the theory, it was "the happiest thought in my life". Perhaps you have heard the story about Newton and the falling apple. I don't know if the story about Newton is true (many think it is not) but there is a similar story about Einstein which he himself narrated. Delivering a lecture in Japan, he said:

> I was sitting in a chair in the patent office at Berne when all of a sudden a thought occurred to me: "If a person falls freely, he will not feel his own weight." I was startled. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation.

The story of the general theory would not be complete without some mention of the experiments done to verify the theory. In fact, it is these experiments which made Einstein famous overnight. The story goes like this. Einstein published his theory in March 1916. Remember Einstein was in Germany at that time, and that a war was going on between England and Germany. Not many people took much notice of Einstein's work because a large number of scientists (on both sides) were actually involved with the war or were preoccupied with the war effort.

One of the predictions of the general theory is that light would be deflected by gravity. The deflection might be small, but is definitely to be expected. A straightforward way of checking this would be to carry out the experiment indicated in Fig. 10.4. It is clear that the experiment cannot be done everyday; rather, one must wait for a total solar eclipse which occurs only once in a while. The English Astronomer Royal, Sir Frank Dyson noted in March 1917 that a total solar eclipse would occur on May 29, 1919 when the theory of Einstein could be put to a test. Though the war was still on, preparations for the experiment had commenced. The British did not worry that Einstein was a German. For scientists, it is science alone that matters.

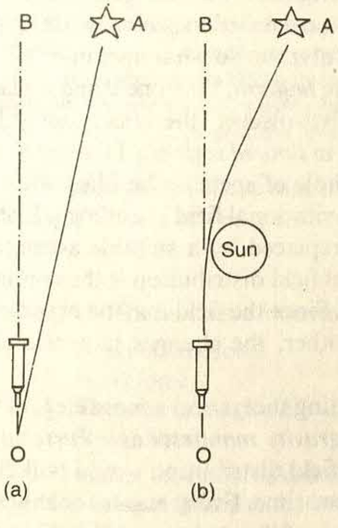


Fig. 10.4 The principle of the light-bending experiment that provided a sensational verification of the general theory is illustrated here. Consider the star A in (a). If the telescope is directed along some other direction like OB, then light from the star would obviously not reach the observer looking through the telescope. However, if there is a massive object (like our Sun) in between, then the light could bend while travelling from the star to the Earth, and the star become visible as illustrated in (b). Such experiments can only be performed during a total solar eclipse for it is only then that the brightness of the Sun does not obscure the star. Much before the expeditions to observe the stars during the eclipse (of 1919), Einstein wrote to his friend Besso, "I do not doubt the correctness of the whole system [i.e., the general theory], whether the observation of the solar eclipse succeeds or not."

Totality of a solar eclipse is observable only in some parts of the world. In this case, it was predicted that it would be seen in Africa and in South America. So the British decided to send two expeditions, one to each continent where the eclipse could be seen. The famous astronomer Eddington was a member of the expedition which went to Africa. (For some interesting material on Eddington, see the companion volume *Chandrasekhar and His Limit*.) The results of the expeditions were presented at a joint meeting of the Royal Society and The Royal Astronomical Society which was held on November 6, 1919. That was a truly historic meeting. Both expeditions reported that observations made by them completely verified the correctness of Einstein's predictions. So light did bend under gravity. Newpapers all over the world flashed the news, and overnight Einstein became a very famous man. It is doubtful if any other scientist has ever enjoyed such fame.

As far as mechanics is concerned, Newton's achievements are two fold. First there are his famous laws of motion and then there is his equally famous law of gravitation. Notice that the two stand apart: But together, they provided a basis for classical celestial mechanics. When Einstein proposed the special theory, there was no reference to gravitation; once again the latter stood apart.

I have earlier described an inertial frame as one on which no forces are acting. Now most of the forces acting on a system can in principle at least be annulled. For example, if electrostatic forces are acting due to the presence of various electric charges, additional charges could be suitably introduced to annul the effect of the original set of charges. But gravitational forces cannot be switched off or cancelled in such a manner. And gravity is present everywhere in the Universe. So what does one do?

Einstein realised that if the field were *uniform*, then one could replace it by an accelerated frame. Unfortunately, however, the gravitational field is never uniform. At best it is so only in *limited* regions. This opens up a possibility. We could imagine the whole of space to be filled with tiny cells such that in each of these the gravitational field is uniform. Each of these uniform fields could then be replaced by a suitable accelerated frame. The non-uniform gravitational field distribution is thus replaced by a collection of accelerated frames. Since the field may be expected to vary gradually from one cell to another, the changes in acceleration would also be gradual.

All this is a very clumsy way of putting things and a more elegant way is indicated in Fig. 10.5. Basically, *gravity manifests as a curvature of space-time*. So the description of the field distribution would boil down to a description of the warping of space-time. Every mass *m* changes the curvature of space-time at the location of that mass, and this curvature diminishes gradually as one travels away from *m*. In short, general relativity describes gravitation in a *geometric* way. Our Earth is curved and yet a football field appears flat. Likewise, curved space-time appears *locally* flat in sufficiently small regions. And in such regions, special relativity applies.

10.4 Unified field theory

Early in the twenties, Einstein started working on a topic which engaged his attention till the very end. This was the *unified field theory*.

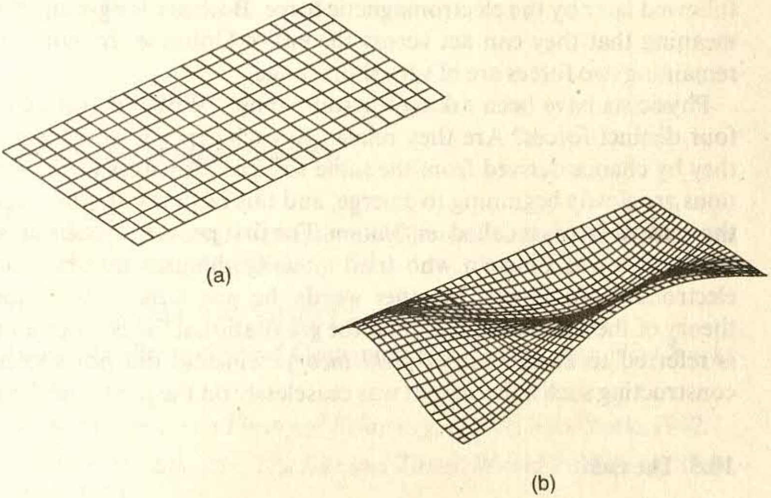


Fig. 10.5 (a) shows a rectangular rubber sheet with a grid marked on it; the sheet is not distorted. (b) shows a similar sheet but which is deformed. Special relativity views space-time as in (a) while general relativity views it as in (b). In other words, in general relativity, space-time is warped, the warping being caused by gravitational field. The larger the strength of this field, the greater is the warping.

These days we believe that there are four fundamental forces (see Box 10.3). The first of these to be discovered was the gravitational force,

Box 10.3 The four basic forces in Nature are:

1. Gravitational force
2. Weak force
3. Electromagnetic force
4. Strong force

In the above, the forces have been arranged according to their strength, the feeblest being at the top of the list.

Physicists now believe that in the early Universe, all these forces were united into one super force, and that they became separated when the Universe started expanding. The first to try and understand this unification was Einstein. As I have explained in the text, he attempted to "marry" the gravitational and the electromagnetic forces. In the sixties, Salam, Weinberg and Glashow succeeded in unifying the electromagnetic force and the weak force into a composite force called the *electroweak* force. Salam has several times acknowledged that he was inspired by Einstein's earlier quest. People are now trying to unite the electroweak force with the strong force and the gravitational force. Whether they will succeed, remains to be seen.

followed later by the electromagnetic force. Both are long-range forces, meaning that they can act across the entire Universe. By contast, the remaining two forces are of very short range.

Physicists have been asking for some time: "Why did Nature create four distinct forces? Are they related to each other in some way? Are they by chance derived from the same source?" Answers to these questions are slowly beginning to emerge, and this business of tying together the various forces is called *unification.* The first person to attempt such a unification was Einstein who tried to unify the gravitational and the electromagnetic forces. In other words, he was looking for a unified theory of the electromagnetic and the gravitational fields. Such a theory is referred to as the *unified field theory.* Einstein did not succeed in constructing such a theory but was ceaselessly on the job till he died.

10.5 The end

For a number of years, Einstein had pain in the abdomen. It was diagnosed as aneurysm, and in 1948 he underwent surgery. There was a relapse in 1954 and in December of that year he was admitted to the hospital. Einstein died on April 18, 1955. Abraham Pais who has written a scientific biography of Einstein describes a visit he made to the famous man just a few days before the latter was admitted to the hospital. Pais went to Einstein's house, and now let us hear the story from him:

> I went upstairs and knocked at the door of Einstein's study. There was his gentle 'Come'. As I entered, he was seated in his arm chair, a blanket over his knees, a pad on the blanket. We spent a pleasant hour or so. Then I told him I should not stay any longer. We shook hands, and I said goodbye. I walked to the door of the study, not more than four or five steps away. I turned around as I opened the door. I saw him in his chair, his pad back on his lap, a pencil in his hand, oblivious to the surroundings. He was back at work.

That shows Einstein's concentration and his dedication.

Einstein continued working even in the hospital, whenever he could. Pais records that the day before he died, Einstein was studying the most recent pages of his calculations on the unified field theory. Einstein probably knew he was dying and was looking at the task which someone else would have to complete. He would be happy to know that some unification *has* now been achieved, though not quite in the manner he foresaw. People who accomplished it make no secret of the inspiration they have derived from Einstein.

Suggestions for further reading

There is a lot of reading material available on Einstein as well as on relativity. Here is a small selection.

1. Bernstein, J. *Einstein*, Viking Press: New York, 1973.
2. Bondi, H. *Relativity and Common Sense*, Doubleday: New York, 1964.
3. Born, M. *Einstein's Theory of Relativity*, Dover: New York, 1962.
4. Clark, R.W. *Einstein, The Life and Times*, World Publishing: New York, 1971.
5. Durell, C.V. *Readable Relativity*, Harper & Row: New York, 1960.
6. Einstein, A. "How I Created the Theory of Relativity" (based on a lecture delivered in Japan in 1922), *Physics Today*, **35**:45, 1982.
7. Einstein, A. *Relativity, the Special and the General Theory*, Crown Publishers: New York, 1961.
8. Geroch, R. *General Relativity from A to B*, University of Chicago Press: Chicago, 1978.
9. Hoffmann, B. *Albert Einstein, Creator and Rebel*, Viking Press: New York, 1972.
10. Hoffmann, B. *Albert Einstein, The Human Side*, Princeton University Press: Princeton, 1979.
11. Levinger, E.E. *Albert Einstein*, Julian Mesner: New York, 1949.
12. Michelmore, P. *Einstein, Profile of the Man*, Dodd, Mead & Co: New York, 1963.
13. Shankland, R.S. "Conversations with Albert Einstein," *American Journal of Physics*, **31**:47, 1963.
14. Taylor, E.F. and Wheeler, J.A. *Spacetime Physics*, W.H. Freeman: San Francisco, 1966.

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G. Venkataraman : At the Speed of Light
ISBN 81 7371 009 0

Rs 60..0